

Moulay Ismail University  
Faculty of Sciences and Technology of Errachidia  
Department of Mathematics

THE INTERNATIONAL CONFERENCE ON ALGEBRA AND ITS  
APPLICATIONS

## Communication Abstracts

FST Errachidia, Morocco

April 26–28, 2017

The Faculty of Sciences and Technology of Errachidia and Department of Mathematics organize an international conference whose theme is :

« The International Conference on Algebra and its Applications »  
Errachidia  
April 26–28, 2017.

The scope of the ICAA-2017 conference encompasses, but is not limited to, the following areas :

- Associative algebra and its applications.
- Commutative algebra and its applications.
- Homological algebra and its applications.
- Number theory and its applications.
- Cryptography and its applications.
- Non-commutative algebra and its applications.
- Applications of algebra to real problems.

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# Preface

We are pleased to introduce the (ICAA 2017) acts, the summarize of abstracts of research that will be presented during the "The International Conference on Algebra And its Applications, Errachidia" on 26-28 April 2017. It is organized by the Faculty of Science and Technology of Errachidia.

The main goal of this conference is to gather the national and international scientific community of the domain to allow the moroccan researchers to exchange and to develop their knowledge of search in the field of algebra and its applications. It also aims to present recent progress and new trends in algebra and its applications. This will allow the participants to enrich their ideas and their knowledge regarding search in this domain. The 167 communications that will be presented can be categorized into variety areas in mathematics which include (but not limited to) :

- Associative algebra and its applications.
- Commutative algebra and its applications.
- Homological algebra and its applications.
- Number theory and its applications.
- Cryptography and its applications.
- Non-commutative algebra and its applications.
- Applications of algebra to real problems.

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# Plenary Speakers

## Generalized permuting $n$ -derivations in prime near-rings

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ABSTRACT. Let  $\mathcal{N}$  be a left near-ring. It is said to be zero-symmetric if  $0x = 0$  holds for all  $x \in \mathcal{N}$ . Recall that in a left near ring  $N$   $x0 = 0$  holds for all  $x \in N$ . Let  $n$  be a positive integer and  $N^n = \mathcal{N} \times \mathcal{N} \times \cdots \times \mathcal{N}$  ( $n$ -copies). A map  $D : N^n \rightarrow \mathcal{N}$  is said to be permuting if the equation  $D(x_1, x_2, \dots, x_n) = D(x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(n)})$  holds for all  $x_1, x_2, \dots, x_n \in \mathcal{N}$  and for every permutation  $\pi \in S_n$ , where  $S_n$  is the permutation group on  $\{1, 2, \dots, n\}$ . An  $n$ -additive (i.e., additive in each argument) mapping  $D : N^n \rightarrow \mathcal{N}$  is called an  $n$ -derivation if

$$D(x_1, x_2, \dots, x_i x'_i, \dots, x_n) = D(x_1, \dots, x_i, \dots, x_n) x'_i + x_i D(x_1, x_2, \dots, x'_i, \dots, x_n)$$

hold for all  $x_1, x_2, \dots, x_i, x'_i, \dots, x_n \in \mathcal{N}$  and for all  $i = 1, 2, \dots, n$ . An  $n$ -additive mapping  $F : N^n \rightarrow \mathcal{N}$  is called a generalized  $n$ -derivation if there exists an  $n$ -derivation  $D$  on  $\mathcal{N}$  such that  $F(x_1, x_2, \dots, x_i x'_i, \dots, x_n) = F(x_1, \dots, x_i, \dots, x_n) x'_i + x_i D(x_1, x_2, \dots, x'_i, \dots, x_n)$  hold for all  $x_1, x_2, \dots, x_i, x'_i, \dots, x_n \in N$ , and for all  $i = 1, 2, \dots, n$ . Moreover, if the map  $D$  (resp.  $F$ ) is permuting, then all the above  $n$  relations are equivalent and  $D$  (resp.  $F$ ) is called permuting  $n$ -derivation (resp. generalized permuting  $n$ -derivation) on  $\mathcal{N}$ . A generalized permuting 1-derivation is a generalized derivation and generalized permuting 2-derivation is a symmetric generalized bi-derivation. The concepts of symmetric bi-derivation was introduced in rings by G. Maksa and subsequently extended to  $n$ -derivation by Park, K.H. and Jung, Y.S., [Commun. Korean Math. Soc. 25, (2010), 1-9]. Motivated by these concepts we have introduced the notions of generalized  $n$ -derivations and generalized permuting  $n$ -derivations in near-rings. Further, these concepts have been extended to  $(\varphi, \psi)$ - $n$ -derivation and generalized  $(\varphi, \psi)$ - $n$ -derivation. Very recently several theorems obtained earlier for derivations and generalized derivations in near-rings have been generalized to  $(\varphi, \psi)$ - $n$ -derivation and generalized  $(\varphi, \psi)$ - $n$ -derivation ( see Ashraf & Aslam [ Georgian Math. J. (2017)]. In the present talk, we give an up-to-date account of the work done by various authors in this direction.

## Class number one problem for splitting fields of some polynomials

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ABSTRACT. Let  $n$  be an integer greater than or equal to 3,  $P(X)$  be an irreducible monic polynomial with coefficients in  $\mathbb{Z}$  and of degree  $n$ ,  $K$  a field generated by a root of  $P(X)$ ,  $L$  the normalizer of  $K$ , and  $d$  be the discriminant of  $P(X)$ . The discriminant  $d$  is equal to

$$d = \prod_{i < j} (\alpha_i - \alpha_j)^2$$

Where the  $\alpha_i$  are the roots of  $P(X)$ . Then  $d$  is a square in  $L$ , so the quadratic field  $F = \mathbb{Q}(\sqrt{d})$  is included in  $L$ . Let  $d(K)$  be the discriminant of  $K$ , then  $F = \mathbb{Q}(\sqrt{d}) = \mathbb{Q}(\sqrt{d(K)})$ . In my conference, I give an algebraic study of the class number one problem for the splitting field  $L$  (the condition for which the class number of  $L$  is equal to 1), in the case where the main condition "  $d(K)$  is not square in  $\mathbb{Z}$  and is equal to the discriminant of  $\mathbb{Q}(\sqrt{d(K)})$ " is satisfied.

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**$n$ -Absorbing ideals of commutative rings and recent progress on  
three conjectures: A survey**

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ABSTRACT. Let  $R$  be a commutative ring with  $1 \neq 0$ . Recall that a proper ideal  $I$  of  $R$  is called a *2-absorbing ideal* of  $R$  if  $a, b, c \in R$  and  $abc \in I$ , then  $ab \in I$  or  $ac \in I$  or  $bc \in I$ . A more general concept than 2-absorbing ideals is the concept of  $n$ -absorbing ideals. Let  $n \geq 1$  be a positive integer. A proper ideal  $I$  of  $R$  is called an  *$n$ -absorbing ideal* of  $R$  if  $a_1, a_2, \dots, a_{n+1} \in R$  and  $a_1 a_2 \cdots a_{n+1} \in I$ , then there are  $n$  of the  $a_i$ 's whose product is in  $I$ . The concept of  $n$ -absorbing ideals is a generalization of the concept of prime ideals (note that a prime ideal of  $R$  is a 1-absorbing ideal of  $R$ ). In this talk, we collect some old and recent results on  $n$ -absorbing ideals of commutative rings.

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## Quelques applications des fonctions $L$ et zêtas

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ABSTRACT. Dans cet exposé, nous étudions les propriétés analytiques et arithmétiques des fonctions zêtas et fonctions  $L$  suivantes :

- (1) Zêta de Riemann, et
- (2) Série et fonctions  $L$  de Dirichlet, et
- (3) Fonctions zêta de Dedekind.

En particulier, nous verrons le théorème de Klengen-Siegel sur la rationalité des valeurs des zêtas de Dedekind associées aux corps de nombres totalement réels. Enfin nous donnons quelques applications.

## Extensions of number fields and $p$ -adic Lie groups

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ABSTRACT. I will explain the philosophy of the theory of Galois representations and of the Fontaine-Mazur Conjecture (FM), specially when the image is potentially everywhere unramified. Then I will give some basic facts concerning  $p$ -adic Lie groups in relation with Galois representations. To conclude, I will give an improvement of a result of Nigel Boston (in 90's) in the unramified context of the FM Conjecture. This is a joint work with Hajir (UMASS).

## About amalgamated algebras along an ideal

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**Dedicated to My Professor El Amin KAIDI**

ABSTRACT. Let  $A$  and  $B$  be two rings with unity, let  $J$  be an ideal of  $B$  and let  $f : A \rightarrow B$  be a ring homomorphism. In this setting, we can consider the following subring of  $A \times B$ :

$$A \bowtie^f J := \{(a, f(a) + j) \mid a \in A, j \in J\}$$

called the amalgamation of  $A$  with  $B$  along  $J$  with respect to  $f$  introduced by M. D'Anna, C. A. Finocchiaro and M. Fontana in 2009. This Talk is a survey about the amalgamation  $A \bowtie^f J$ .

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## Post quantum cryptography

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ABSTRACT. A quantum computer with Shor's algorithm will solve the integer factorization and the discrete logarithm problems upon which most of the widely used cryptosystems such as RSA (Rivest, Shamir, Adleman) and ECC (elliptic curve cryptography) are based. Nevertheless, some cryptosystems running on conventional computers, such as NTRU, LWE and McEliece are still resisting to quantum computers. Such cryptosystems are good candidates for post quantum cryptography. In this talk, we will present the most promising post quantum cryptosystems and discuss their security.

## Some commutativity theorems in rings with involution

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ABSTRACT. In this paper we investigate commutativity of ring  $R$  with involution  $*$  which admits a derivation satisfying certain algebraic identities. Some well-known results characterizing commutativity of prime rings have been generalized. Finally, we provide examples to show that various restrictions imposed in the hypotheses of our theorems are not superfluous.

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# Speakers

## Realizability of linear recurrence sequences

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ABSTRACT. A sequence of non-negative integers  $(u_n)_{n \geq 1}$  is called exactly realizable if there is a set  $X$  and a map  $T : X \rightarrow X$  such that  $u_n = |Per_n T|$ , That means  $T$  has exactly  $u_n$  points of period  $n$ . A combinatorial device gives necessary and sufficient conditions for a sequence of non-negative integers to counts the periodic points in a dynamical system. This is applied to study linear recurrence sequences which count periodic point.

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## Efficient presentations of finite 2-groups associated to a pro-2-group with coclass 3

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ABSTRACT. There are 5 infinite pro-2-groups of coclass at most 3 and trivial Schur multiplier. Three of them are metacyclic. One of the non-metacyclic pro-2-groups with trivial Schur multiplier and coclass 3 is

$$S = \langle a, u \mid a^2 = u^4, (u^2)^a = u^{-2} \rangle.$$

This pro-2-group has 5 infinite families of finite 2-groups, three of them contains finite 2-groups with trivial Schur multiplier and two not. In this paper we write down efficient presentations of three of these 5 infinite families.

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## Study of the commutativity of certain rings with involution

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ABSTRACT. In this paper we investigate some commutativity criterions for a ring with involution in which generalized derivations satisfy certain algebraic identities. Moreover, we provide examples to show that the assumed restriction cannot be relaxed.

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## Unitar multiplicatively perfect numbers and their generalizations

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ABSTRACT. Unitary divisors (called block factors) were first considered by R. Vaidyanathaswamy [5]. The current terminology was introduced by E. Cohen [2, 3]. Let the function  $T^*(N)$  denote the product of all unitary divisors of  $N$ . A natural number  $N$  is called multiplicatively unitary  $k$ -perfect if  $T^*(N) = N^k$ . In [4], Sa'ndor introduced and characterized multiplicatively perfect numbers. The relevant results for multiplicatively unitary perfect numbers were obtained by Bege in [6]. In this paper we consider a wider set of multiplicatively unitary perfect numbers and we give a characteristic property for these numbers.

In 1971 (Peter Hags [1]) introduced the concept of the *unitary amicable* as follows two positive integers are said to unitary amicable if the sum of the unitary divisors of each is equal to their sum. In this paper we introduced the concept of *unitary multiplicatively amicable* and study some relevant properties.

A positive number that is greater(less) than the sum of all positive integers that are submultiples of it is called deficient(abundant). In this paper we also present the *unitary multiplicatively abundancy index* as a new tool to study unitary multiplicatively perfect numbers.

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## On finite groups in which semipermutability is a transitive relation

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ABSTRACT. Let  $G$  be a finite group and let  $H$  be a subgroup of  $G$ .  $H$  is said to be semipermutable in  $G$  if  $H$  permutes with every subgroup  $K$  of  $G$  with  $(|H|; |K|) = 1$ . A number of new characterizations of finite solvable  $BT$ -groups are considered, where a  $BT$ -group is one in which semipermutability is a transitive relation.

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**On root-involutions and root-subgroups of certain Chevalley  
groups over finite fields of even characteristic**

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ABSTRACT. In this talk we give a construction of certain Chevalley groups of type  $E_6$  using their root-involutions and root-subgroups.

## Groups with primes order classes

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ABSTRACT. The order classes of a finite group  $G$  denoted by  $OC(G)$ , can give certain and valuable properties of the group. Unfortunately, not many research illustrate the order classes of finite groups, it is correspond to many factors, such as the exponent of the group. Also the order of the group itself. Even that, the previous research determined the order classes for certain groups. Where the group structure showed these classes. On the contrary, this paper aims to configure some groups using their order classes. This will introduce a new notion in finite groups called POC-group "Primes Order Classes group". That is: A POC-group  $G$  is a finite group in which each  $i \in OC(G)$  is a prime factor of  $|G|$ .

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## Additive mappings in prime and semiprime rings with involution

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ABSTRACT. Let  $R$  be an associative ring with center  $Z(R)$ . For every associative ring  $R$  can be turned into a Lie ring by introducing a new product  $[x, y] = xy - yx$ , known as Lie product. So we may regard  $R$  simultaneously as an associative ring and as a Lie ring. A function  $f : R \rightarrow R$  is called a  $*$ -centralizing on  $R$  if  $[f(x), x^*] \in Z(R)$  holds for all  $x \in R$ . In the special case where  $[f(x), x^*] = 0$  for all  $x \in R$ ,  $f$  is said to be  $*$ -commuting on  $R$ .

In this talk, we will discuss the recent progress made on the topic and related areas. Moreover, some examples and counter examples will be discussed for questions raised naturally.

## On the centralizer of generators in 3-Braid group

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(With A. Riaz and M. Arshad)

ABSTRACT. Let  $A$  and  $B$  be two rings,  $J$  an ideal of  $B$  and  $f : A \rightarrow B$  a ring homomorphism. The ring  $A \bowtie^f J := \{(a, f(a) + j \mid a \in A, \text{ and } j \in J\}$  is called the amalgamation of  $A$  with  $B$  along  $J$  with respect to  $f$ . It was proposed by D'anna and Fontana, as an extension for the Nagata's idealization. In this paper we establish necessary and sufficient conditions under which  $A \bowtie^f J$ , and some related constructions, is either a Hilbert ring, a  $G$ -domain or a  $G$ -ring in the sense of Adams. By the way, we investigate  $G$ -ideals in the trivial ring extensions and amalgamations of rings. Our results provide original illustrating examples.

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Mathematics Subject Classification: 20F10, 20F36, 20M05, 57M25.

Key Words and phrases: braid, Grobner-Shirshov basis, centralizer, normal form, summit word, quasipositive braid.

## On the near-common neighborhood graph of a graph

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ABSTRACT. The near common-neighborhood graph of a graph  $G$ , denoted by  $ncn(G)$ , is the graph on the some vertices of  $G$ , tow vertices being adjacent if there is at least one vertex in  $G$  not adjacent to both of them. A graph is called near-common neighborhood graph if it is the near-common neighborhood of some graph. In this paper we introduce the near-common neighborhood of a graph, the near common neighborhood graph, near-completeness number of a graph, basic properties of these new graphs are obtained and interesting results are established.

## Types and uncountable orderings

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ABSTRACT. We shall survey results about directed sets of uncountable size presenting a definite classification.

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## Commutative rings and modules that are $\text{nil}_*$ -coherent or special $\text{nil}_*$ -coherent

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(With D. E. Dobbs and N. Mahdou)

ABSTRACT. Recently, Xiang and Ouyang defined a (commutative unital) ring  $R$  to be  $\text{Nil}_*$ -coherent if each finitely generated ideal of  $R$  that is contained in  $\text{Nil}(R)$  is a finitely presented  $R$ -module. We define and study  $\text{Nil}_*$ -coherent modules and special  $\text{Nil}_*$ -coherent modules over any ring. These properties are characterized and their basic properties are established. Any coherent ring is a special  $\text{Nil}_*$ -coherent ring and any special  $\text{Nil}_*$ -coherent ring is a  $\text{Nil}_*$ -coherent ring, but neither of these statements has a valid converse. Any reduced ring is a special  $\text{Nil}_*$ -coherent ring (regardless of whether it is coherent). Several examples of  $\text{Nil}_*$ -coherent rings that are not special  $\text{Nil}_*$ -coherent rings are obtained as byproducts of our study of the transfer of the  $\text{Nil}_*$ -coherent and the special  $\text{Nil}_*$ -coherent properties to trivial ring extensions and amalgamated algebras.

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Key words: Coherence, nilradical,  $\text{Nil}_*$ -coherence, special  $\text{Nil}_*$ -coherence, finitely presented module, amalgamated algebra, trivial extension.

**On generalized quadrangles of types  $\overline{O}_6(2)$**

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ABSTRACT. The purpose of this talk is to discuss certain geometric properties of generalized quadrangles  $(\Omega, \mathcal{L})$  of type  $\overline{O}_6(2)$ , and show how these properties can be used to construct a Lie algebra of type  $E_6(K)$  for fields of characteristic 2.



## On a conjecture of Franz Lemmermeyer

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(With M.C. Ismaili and M.Talbi)

ABSTRACT. , Recently Franz Lemmermeyer made a conjecture about the 3-class groups of certain pure cubic fields and their normal closures. This paper proves his conjecture and give a counterexamples of one case of his conjecture.

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## On an extension of Serre's Theorem

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ABSTRACT. The vanishing theorem of Serre says that if  $\mathcal{F}$  is a quasi-coherent sheaf on an affine scheme  $X$  then for any  $i > 0$  we have  $H^i(X, \mathcal{F}) = 0$ . In this work we want to weaken the condition "  $X$  is affine" by introducing a property on the sheaf  $\mathcal{F}$ . In particular we discover some criterion for affineness.

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## Two-sided residuation on topologizing filters on commutative rings

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(With J. van Den Berg)

ABSTRACT. The set  $FilR_R$  of all right topologizing filters on a fixed but arbitrary ring  $R$  is both a complete lattice under inclusion, and a monoid with respect to an order compatible, but in general noncommutative binary operation  $:$ . It is known that the order dual  $[FilR_R]^{du}$  of  $FilR_R$  is always left residuated, meaning, for each pair  $\mathfrak{F}, \mathfrak{G} \in FilR_R$  there exists a smallest filter  $\mathfrak{H} \in FilR_R$  such that  $\mathfrak{H} : \mathfrak{G} \supseteq \mathfrak{F}$ , but is not, in general, right residuated (there exists a smallest filter  $\mathfrak{H}$  such that  $\mathfrak{G} : \mathfrak{H} \supseteq \mathfrak{F}$ ).

The binary operation  $:$  is defined on  $[FilR_R]^{du}$  as follows:

$$\mathfrak{F} : \mathfrak{G} = \{K \leq R_R : \exists H \in \mathfrak{F} \text{ such that } K \subseteq H \text{ \& } h^{-1}K \in \mathfrak{G} \forall h \in H\}.$$

Thus the order dual  $[FilR_R]^{du}$  of  $FilR_R$  has the structure of a lattice ordered monoid.

The importance of the structure  $FilR_R$  (as a tool for analysing the ring  $R$ ), lies in the fact that it encodes at least as much information about the ring  $R$  as does the ideal lattice  $IdR$ , for there is a canonical structure preserving embedding (that is in general not onto) of  $IdR$  into  $[FilR_R]^{du}$  that takes each  $I \in IdR$  onto the set of all right ideals of  $R$  containing  $I$ . However, whereas  $IdR$  enjoys residuation on both sides,  $[FilR_R]^{du}$ , is in general, left but not right residuated.

It has been shown in [3] that for every right fully bounded noetherian ring  $R$ ,  $[FilR_R]^{du}$  is two-sided residuated and that a valuation domain will be too if and only if it is rank one discrete. If  $R$  is any ring for which (the monoid operation  $:$  on)  $FilR_R$  is commutative, then obviously  $[FilR_R]^{du}$  is two-sided residuated.

The purpose of this paper is to show that the converse is true whenever the ring  $R$  is commutative. That is, if  $R$  is a commutative ring for which  $[FilR_R]^{du}$  is two-sided residuated, then  $FilR_R$  is commutative, that is to say,  $\mathfrak{F} : \mathfrak{G} = \mathfrak{G} : \mathfrak{F} \forall \mathfrak{F}, \mathfrak{G} \in FilR_R$ . We also provide several non-torsion theoretic characterizations of the two-sided residuated property for a commutative ring  $R$  which show that the two-sided residuation property is equivalent to the conditions that the factor ring  $R/I$  satisfies finiteness conditions (ACC and DCC) on annihilator ideals and the right  $R$ -module  $(R/I)_R$  satisfies finiteness conditions (ACC and DCC) on hereditary pretorsion submodules for all proper ideals  $I$  of  $R$ . We also proved that for a commutative semiartinian ring, artinianness is not a necessary condition for  $FilR_R$  to be commutative.

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## A note on prime ring with generalized skew derivations

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ABSTRACT. Let  $R$  be a prime ring,  $Q_r$  be the right Martindale quotient ring and  $C$  be the extended centroid of  $R$ . For any automorphism  $\varphi$  of  $R$ , an additive mapping  $\delta : R \rightarrow R$  is said to be a  $\varphi$ -derivation or skew derivation of  $R$  with respect to  $\varphi$  if its satisfy  $\delta(xy) = \delta(x)y + \varphi(x)\delta(y)$  for all  $x, y \in R$ . The standard identity  $s_4$  in four variables is defined by  $s_4 = \sum (-1)^\sigma X_{\sigma(1)}X_{\sigma(2)}X_{\sigma(3)}X_{\sigma(4)}$ , where  $(-1)^\sigma$  is the sign of a permutation  $\sigma$  of the symmetric group of degree 4. An additive mapping  $\mathcal{G} : R \rightarrow R$  is a generalized skew derivation if  $\mathcal{G}(xy) = \mathcal{G}(x)y + \varphi(x)\delta(y)$  for all  $x, y \in R$ , where  $\delta$  is an associated skew derivation of  $\mathcal{G}$  and  $\varphi$  is an associated automorphism of  $\mathcal{G}$ . A skew derivation  $(\delta, \varphi)$  of  $R$  is called  $Q$ -inner if its extension to  $Q$  is inner, that is, there exists  $q \in Q$  such that  $\delta(x) = \varphi(x)q - qx$  for all  $x \in Q$ , and otherwise it is  $Q$ -outer. Analogously, an automorphism  $\varphi$  of  $R$  is called inner, if when acting on  $Q$ ,  $\varphi(x) = gxg^{-1}$  for some invertible element  $g \in Q$ . When  $\varphi$  is not inner, then it is called an outer automorphism. An automorphism  $\varphi$  of  $Q$  is called Frobenius, if in the case of  $\text{char}(R) = 0$ ,  $\delta(\lambda) = \lambda$  for all  $\lambda \in C$  and if, in the case of  $\text{char}(R) = p \geq 2$ ,  $\varphi(\lambda) = \lambda^{p^n}$  for all  $\lambda \in C$ , where  $n$  is a fixed integer, positive, zero, or negative. In the present talk we discuss the behaviour of generalized skew derivation acting on multilinear polynomials in prime rings and obtain a description of the structure of  $R$  and information on the form of  $\mathcal{G}$  in terms of  $s_4$  and the multiplication by a specific element from the extended centroid of  $R$ .

## Epimorphically preserved semigroup identities

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ABSTRACT. It is shown that a particular classes of semigroup identities whose both sides contain repeated variables and are preserved under epis in conjunction with all seminormal identities.

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## ***w*-Modules over commutative rings**

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ABSTRACT. Let  $R$  be a commutative ring and let  $M$  be a GV-torsionfree  $R$ -module. Then  $M$  is said to be a  $w$ -module if  $\text{Ext}_R^1(R/J, M) = 0$  for any  $J \in \text{GV}(R)$ , and the  $w$ -envelope of  $M$  is defined by  $M_w = \{x \in E(M) \mid Jx \subseteq M \text{ for some } J \in \text{GV}(R)\}$ .

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## On countably generated extensions of $QTAG$ -modules

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ABSTRACT. Suppose  $M$  is a  $QTAG$ -module with a submodule  $K$  such that  $M/K$  is countably generated that is, in other words,  $M$  is a countably generated extension of  $K$ . A problem of some module-theoretic interest is that of whether  $K \in \mathcal{F}$ , a class of  $QTAG$ -modules, does imply that  $M \in \mathcal{F}$ . The aim of the present article is to settle the question for certain kinds of modules, when  $\mathcal{F}$  coincides with the class of all totally projective  $QTAG$ -modules.

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Mathematics Subject Classification (2010): 16 K 20.

Key words:  $\Sigma$ -modules,  $\sigma$ -summable modules, summable modules,  $\alpha$ -modules,  $(\omega + n)$ -projective modules.

## Solving the fuzzy polynomial equations by Fuzzy structured element method

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(With S. Melliani)

ABSTRACT. In this paper, we define the fuzzy polynomial equation with fuzzy points, we investigate the resolution of fuzzy polynomial equations based on the fuzzy structured element, and we propose the method transforming the fuzzy polynomial equations into the parametric equations.

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## On divided and regular divided rings

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**Dedicated to My Professor El Amin KAIDI.**

ABSTRACT. In this paper, we study the notion of divided and regular divided rings. Then we establish the transfer of these notions to trivial ring extension and amalgamated algebras along an ideal. These results provide examples of non-divided regular divided rings. The article includes a brief discussion of the scope and precision of our results.

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**On 27-dimensional modules of type  $E_6(K)$ , for fields  $K$  of characteristic two**

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ABSTRACT. The aim of this talk is to give an elementary and self-contained construction of Lie algebras  $E_6(K)$  of type  $E_6$  over finite fields  $K$  of characteristic 2. If  $V$  is a 6-dimensional vector space over  $\mathbb{F}_2$  with a non-degenerate quadratic bilinear form  $Q$  on  $V$  of minimal Witt-index, then the pair  $(\Omega, \mathcal{L})$  is a generalized quadrangle of type  $O_6^-(2)$  where  $\Omega = \{0 \neq x \in V \mid Q(x) = 0, \dim X = 2\}$  and  $\mathcal{L} = \{X \leq V \mid Q(X) = 0, \dim X = 2\}$

The construction is mainly based on the notion of M-sets  $\Delta$  introduced in [1] as a root-system of  $E_6$  and on the root-elements  $M_\Delta \in \text{End}_K(A)$  where  $A$  is a 27-dimensional vector space over  $\mathbb{F}_2$  with basis  $e_x, x \in \Omega$  and  $\begin{cases} e_x, x \notin \Delta \\ e_x \sigma_x, x \in \Delta \end{cases}$  and  $\sigma_s$  is the reflection map adjoint with  $\Delta$ .

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## Transcendence and measure of transcendence of continued fractions

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ABSTRACT. In this article, we give sufficient conditions on the continued fractions  $A$  and  $B$  so that the real numbers  $A$ ,  $B$ ,  $A \pm B$ ,  $AB$ ,  $A/B$  can be transcendental. The used method also permits us to calculate the measure of transcendence as well as a measure of algebraic independence of transcendental continued fractions.

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## **Hopficité des modules (Survey)**

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RÉSUMÉ. Dans ce travail on donne les différentes notions de la hopficité des modules, les principales propriétés, les différents résultats de tels modules et ses relations avec d'autres classes de modules plus larges.

## An efficient image encryption technique based ECC and DNA computing

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ABSTRACT. Due to the rapid growth of digital communication and multimedia application, security becomes an important issue in data exchange process over the wide network. Recently, Image encryption gained a lot of attention in the field of protection of multimedia data like images. In this paper, a new encryption image algorithm with two levels of security is proposed. The first level of security is provided by encoding the original image using DNA Computing technique. Further, the second level of security is provided by using ECC encryption and decryption algorithm. The novelty of the proposed method is advantages of both ECC and DNA computation is exploited in providing a high level of data security. Finally, the paper explains in detail the implementation of the proposed method using MATLAB R2015a.

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Mathematics Subject Classification (2010): .

Key words: Cryptography, Elliptic Curve, plain Image, Deoxyribo Nucleic Acid (DNA), encryption, Decryption.

## On the unimodality of the open-set polynomials

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ABSTRACT. Let  $\tau$  be a topology on the finite set  $X_n$ . We consider the *open-set polynomial* associated with the topology  $\tau$ . Its coefficients  $a_k$  are the numbers of open sets of size  $k = 0, \dots, n$ .

If the topology has a large number of open sets, then its open-set polynomial is determined explicitly and shown to be unimodal. In passing, we prove that this polynomial has real zeros only in the case where  $\tau$  is the discrete topology. This answers a question raised by J. Brown.

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Mathematics Subject Classification (2010): .

Key words: .

The first part of this work was done when the author was at Alimam university, Riyadh. K.S.A.

## Synchronization of a chaotic system by generalized active control

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ABSTRACT. This paper designs a scheme for controlling a chaotic system to a period system using active control technique. We have discussed about the synchronization scheme between two identical coupled chaotic systems (Four-scroll attractor ) via active control. Numerical Simulation results are presented to show the effectiveness of the proposed scheme.

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## On the discriminator of binary recurrent sequences

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ABSTRACT. Given  $k \geq 1$  consider the recurrent sequence determined by  $u_k(n+2) = (4k+2)u_k(n+1) - u_k(n)$ , with initial values  $u_k(0) = 0$ ,  $u_k(1) = 1$ . For  $n = 0, 1, 2, 3, \dots$  the discriminator function  $\mathcal{D}_k(n)$  of  $u_k(n)$  is defined as the smallest integer  $m$  such that  $u_0, u_1, \dots, u_{n-1}$  are pairwise incongruent modulo  $m$ . Put  $\Delta = (4k+2)^2 - 4 > 0$  and  $\mathcal{P}_\Delta := \{p : p \mid \Delta\}$ . We prove that, for all  $k = 2, \dots, 6$ , the value of the discriminator  $\mathcal{D}_k(n)$  is the sequence of the smallest integer greater or equal to  $n$  with prime factors only in  $\mathcal{P}_\Delta$ . We also classify all binary recurrent sequence  $\{u_n\}_{n \geq 0}$  consisting of different integer terms such that  $\mathcal{D}_u(2^e) = 2^e$  for  $e \geq 1$ . This is a joint work with Pieter Moree from Max Planck Institute for Mathematics.



## ***S*-Noetherian property for noncommutative rings**

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ABSTRACT. In [6], D. D. Anderson and T. Dumitrescu defined *S*-Noetherian rings for commutative rings with identity as a generalization of Noetherian rings and proved a generalization of Cohen's well-known theorem: A commutative ring with identity is Noetherian if and only if each of its prime ideals is finitely generated, [6]. In [1] and [2], Cohen's Theorem is extended to noncommutative context using completely prime right ideals and Oka families of right ideals. In this work, we define right *S*-Noetherian property for noncommutative rings and examine some properties of right *S*-Noetherian rings. We apply the facts on Oka families of right ideals to generalize Cohen's Theorem to right *S*-Noetherian rings. We use Oka families of right ideals and point annihilator sets to give two characterizations of right *S*-Noetherian rings in terms of completely prime right ideals.

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Mathematics Subject Classification (2010): 16D25, 16D80.

Key words: Right *S*-Noetherian rings, completely prime right ideals, Oka families of right ideals, point annihilator sets.

## On $pm^+$ and finite character bi-amalgamation

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ABSTRACT. Let  $f : A \rightarrow B$  and  $g : A \rightarrow C$  be two ring homomorphisms and let  $J$  and  $J'$  be two ideals of  $B$  and  $C$ , respectively, such that  $f^{-1}(J) = g^{-1}(J')$ . The bi-amalgamation of  $A$  with  $(B, C)$  along  $(J, J')$  with respect of  $(f, g)$  is the subring of  $B \times C$  given by

$$A \bowtie^{f,g} (J, J') = \{(f(a) + j, g(a) + j') / a \in A, (j, j') \in J \times J'\}$$

In this paper, we study the transference of  $pm^+$ ,  $pm$  and finite character ring-properties in the bi-amalgamation.

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## On spaces of topological complexity two

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ABSTRACT. In this talk I consider the rational interpretation of results about the classification of minimal cell structures of spaces of topological complexity two under some hypotheses on their graded cohomological algebra. This continuous method used by M.Grant et al. in [5].

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Mathematics Subject Classification (2010): .

Key words: topological complexity, sectional category, cup length.

## Geometry of quiver

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ABSTRACT. The aim of the representation theory of algebras is to understand the category of modules over a given associative unital  $k$ -algebra  $A$ , where  $k$  is a commutative ring. In this talk, I will restrict to the case that  $A$  is finite-dimensional over an algebraically closed field  $k$ , and I will focus on the geometrical aspects of representations of quivers.

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## Inner local spectral radius preservers of operator products

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(With M. E. El Kettani and H. Benbouziane)

ABSTRACT. Let  $\mathcal{B}(X)$  be the algebra of all bounded linear operators on an infinite dimensional complex Banach space  $X$ . For an operator  $T \in \mathcal{B}(X)$  and a vector  $x \in X$ , let  $\iota_T(x)$  denote the inner local spectral radius of  $T$  at  $x$ . For  $x_0 \in X$  nonzero fixed vector we determine the form of surjective maps preserving the inner local spectral radius of  $TS + R$  for all  $T, S$  and  $R \in \mathcal{B}(X)$ . We characterize also maps  $\varphi$  from  $\mathcal{B}(X)$  into itself satisfying  $\iota_{\varphi(T)\varphi(S)}(x) = \iota_{TS}(x)$  for all  $T, S \in \mathcal{B}(X)$  and  $x \in X$ .

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## Quasipolarity of a ring with respect to jacobson radical

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(With S. Halicioglu and A. Harmanci)

ABSTRACT. In this paper, we introduce a class of  $J$ -quasipolar rings. Let  $R$  be a ring with identity. An element  $a$  of a ring  $R$  is called *weakly  $J$ -quasipolar* if there exists  $p^2 = p \in \text{comm}^2(a)$  such that  $a + p$  or  $a - p$  are contained in  $J(R)$  and the ring  $R$  is called *weakly  $J$ -quasipolar* if every element of  $R$  is weakly  $J$ -quasipolar. We give many characterizations and investigate general properties of weakly  $J$ -quasipolar rings. If  $R$  is a weakly  $J$ -quasipolar ring, then we show that (1)  $R/J(R)$  is weakly  $J$ -quasipolar, (2)  $R/J(R)$  is commutative, (3)  $R/J(R)$  is reduced. We use weakly  $J$ -quasipolar rings to obtain more results for  $J$ -quasipolar rings. We prove that the class of weakly  $J$ -quasipolar rings lies between the class of  $J$ -quasipolar rings and the class of quasipolar rings. Among others it is shown that a ring  $R$  is abelian weakly  $J$ -quasipolar if and only if  $R$  is uniquely clean.

## A generalization of $j$ -quasipolar rings

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(With S. Halicioglu and A. Harmanci)

ABSTRACT. In this paper, we introduce a class of quasipolar rings which is a generalization of  $J$ -quasipolar rings. Let  $R$  be a ring with identity. An element  $a \in R$  is called  $\delta$ -quasipolar if there exists  $p^2 = p \in \text{comm}^2(a)$  such that  $a + p$  is contained in  $\delta(R)$ , and the ring  $R$  is called  $\delta$ -quasipolar if every element of  $R$  is  $\delta$ -quasipolar. We use  $\delta$ -quasipolar rings to extend some results of  $J$ -quasipolar rings. Then some of the main results of  $J$ -quasipolar rings are special cases of our results for this general setting. We give many characterizations and investigate general properties of  $\delta$ -quasipolar rings.

## Some homological properties of amalgamated duplication of a ring along an ideal

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Dedicated to My Professor El Amin KAIDI

ABSTRACT. We investigate the transfer of some homological properties from a ring  $R$  to his amalgamated duplication along some ideal  $I$  of  $R$   $R \bowtie I$ , and then generate new and original families of rings with these properties.

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## Generalized semi-derivations and generalized left semi-derivations of prime rings

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ABSTRACT. In that paper there is explored the commutativity of a prime ring in which generalized semi-derivations satisfy certain differential identities. Furthermore, we have introduced the notion of generalized left semi-derivations in a noncommutative ring  $R$  and the main results state some generalizations of recent results due to Chan, Jun, Jung and Firat.

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Mathematics Subject Classification (2010): 16W10, 16W25, 16U80.

Key words: ring, semi-derivations, left semi-derivations, generalized semi-derivations, left generalized semi-derivations.

## Hydrological modeling and geotechnologies for analysis of susceptibility to floods and flash floods in places with low availability of altimetric and hydrological data: the case of the South Brazilian Forqueta River Basin

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**ABSTRACT.** Natural disasters very often have caused deep impacts on human society. It is estimated that in the last decade about 1.3 billion people around the globe have been affected by hydrometeorological events. Considering the limited investments in hydrological monitoring and topographical surveys in Brazil, it becomes necessary the development of alternative methods to the traditional identification of susceptible areas to floods. The main purpose of this study is to present an approach to the identification of susceptible areas to floods, adapted to regions with low availability of data, integrating hydrological models and geotechnologies with the use of free data available in a large scale. The study was applied to the Forqueta River Basin, which has been affected by an intense flood in 2010. The identification of susceptible areas to floods in the Forqueta River Basin has been conducted by the utilization of a modelling method that integrates the hydrological simulation and the use of geotechnologies. The definition of extreme rainfall in return periods (RP) of 10, 30 and 100 years was carried out by using an intensity-duration-frequency equation. Shuttle Radar Topography Mission data were used to the delimitation of the basins and rivers. The Soil Conservation Service method was used for the transformation of rain runoff, while the spread of river flood wave was conducted by Muskingum-Cunge model. The extreme precipitation scenario for a period of 25 hours ranged from 123mm (RP10) to 179mm (RP100). Hydrological simulation revealed that the maximum flow rates can exceed  $8.000m^3.s^{-1}$  at the Forqueta River outfall, with wet area section larger than  $5.000m^2$  and waters rising more than 10m on the average. About 2% of the basin showed some degree of susceptibility (RP100), adding up to  $53km^2$  of wetlands. The approach has shown consistent results concerning to flows and flood levels. One may conclude that the same data set could be applied to other fields of study, for regional hydrological characterization of susceptibility to disasters.

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Key words: geographic information systems, digital elevation model, natural disasters, mathematical modeling, environmental sciences.

## The non-commutative geometry on the compactification of matrix model

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ABSTRACT. We review briefly the main lines of the toroidal compactification of the IKKT model. This study, which is also valid for the BFSS matrix theory, allow us to give a reformulation of the defining constraint equation of Banks et al, useful when we showing the compactification on  $S_2$  and  $F_0$ .

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Mathematics Subject Classification (2010): .

Key words: BFSS and IKKT models, Compactification, Matrix Model, Non- Commutative geometry.

## Construction of linear codes related to faithful representations of simple Lie algebras

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(With M. Ait Ben Haddou and M. Najmeddine)

ABSTRACT. Using the finite-dimensional faithful representations of simple Lie algebras, we construct linear codes over the ring of integers. We exemplify the construction for codes related to the adjoint representation of simple Lie algebras of type  $A_n$ ,  $B_n$ ,  $C_n$ ,  $D_n$ ,  $E_6$ ,  $E_7$ ,  $E_8$ ,  $F_4$  and  $G_2$ . In determining the minimal distance of this codes, we have used the generator matrix.

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## On the stability of $(\alpha, \beta, \gamma)$ -derivations on Lie algebras

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ABSTRACT. A Lie algebra  $A$  is an algebra endowed with the Lie product

$$[x, y] = xy - yx.$$

A  $\mathbb{C}$ -linear mapping  $D : A \rightarrow A$  is called a Lie derivation of  $A$  if  $D : A \rightarrow A$  satisfies

$$D[x, y] = [D(x), y] + [x, D(y)]$$

for all  $x, y \in A$ . Following a  $\mathbb{C}$ -linear mapping  $D : A \rightarrow A$  is called a  $(\alpha, \beta, \gamma)$ -derivation of  $A$  if there exist  $\alpha, \beta, \gamma \in \mathbb{C}$  such that

$$\alpha D[x, y] = \beta[D(x), y] + \gamma[x, D(y)]$$

for all  $x, y \in A$ . For more details see [1] and [12].

In this talk, we consider the stability of the following  $(\alpha, \beta, \gamma)$ -derivation equation

$$\alpha D[x, y] = \beta[D(x), y] + \gamma[x, D(y)]$$

associated to the  $(m, n)$ -Cauchy Jensen type functional equation

$$\sum f\left(\frac{\sum x_{i_j}}{m} + \sum x_{k_j}\right) = \frac{n-m+1}{n} \left[\sum g(x_i)\right]$$

for all  $x_{i_j} \in A$  in Lie algebras.

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## Weakly coherent property in amalgamated algebra along an ideal

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ABSTRACT. Let  $f : A \rightarrow B$  be a ring homomorphism and let  $J$  be an ideal of  $B$ . In this paper, we investigate the weakly coherent property that the amalgamation  $A \bowtie_f J$  might inherit from the ring  $A$  for some classes of ideals  $J$  and homomorphisms  $f$ . Our results generate original examples which enrich the current literature with new families of examples of non-coherent weakly coherent rings.

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Mathematics Subject Classification (2010): 13F05, 13A15, 13B10, 13D02, 13D05.

Key words: Weakly coherent ring, coherent ring, amalgamated duplication, amalgamated algebra.

## **An application of Linear algebra to image compression**

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(With H. Aksasse, M. Ouhda, B. Aksasse and M. Ouanan)

**ABSTRACT.** In these recent decades, the important and fast growth in the development and demand of multimedia products is contributing to an insufficient in the bandwidth of device and network storage memory. Consequently, the theory of data compression becomes more significant for reducing the data redundancy in order to save more transfer and storage of data. In this context, this paper addresses the problem of the lossless and the near-lossless compression of images. This proposed method is based on Block SVD Power Method that overcomes the disadvantages of Matlab's SVD function. The experimental results show that the proposed algorithm has a better compression performance compared with the existing compression algorithms that use the Matlab's SVD function. In addition, the proposed approach is simple and can provide different degrees of error resilience, which gives, in a short execution time, a better image compression.

## On the structure of some finitely generated $R[G]$ -modules

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ABSTRACT. Let  $G$  be a finite solvable group and  $(R, \pi R, k)$  a SPI-ring where  $k$  its residual field. In this work, we expose some new results about the structure of some finitely generated  $R[G]$ -modules. More precisely, we show that if the characteristic  $p$  of  $k$  does not divide the order  $n$  of  $G$ , then for any finitely generated  $R[G]$ -module  $M$  of finite length such that  $\tilde{G}M = 0$  - there exist a unique  $R$ -module  $N$  such that  $M$  is isomorph - as  $R$ -module- to an external direct sum of  $f_p(n)$  copies of  $N$  where  $f_p(n)$  is the greatest common divisor of all orders of  $p$  modulo  $q$  where  $q$  runs in the set of all prime divisors of  $n$ . In other hand we show that if the residual field  $k$  of  $R$  is a finite field and  $M$  is a finitely generated  $R[G]$ -module  $M$  of finite length such that  $\tilde{G}M = 0$ , then the  $p$ -adic valuation of the order of  $M$  is a multiple of the greatest common divisor  $d_p(n)$  of all orders of  $p$  modulo  $d$  where  $d$  runs in the set of all divisors  $d \geq 2$  of  $n$ . In other words the order of  $M$  is a power  $q^m$  of  $q$  where  $q$  is the order of the residual field  $k$  and  $m$  is an integer such that  $m \geq 2$ . As an application we give an upper bound of the  $p$ -part (the  $p$ -Sylow subgroup)  $Cl_p(L/K)$  of the relative class group  $Cl(L/K)$  of a relative Galois extension of numbers fields  $L/K$ .



## Retractability and co-retractability and properties of endomorphism ring

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ABSTRACT. Let  $R$  be a ring with unity,  $M$  a left  $R$ -module.  $M$  is said to be retractable (resp co-retractable) if for every nonzero  $R$ -submodule  $N$ , there exists a nonzero endomorphism  $u$  of  $M$  such that  $u(M)$  is contained in  $N$  ( resp for every nonzero proper  $R$ -submodule  $N$ , there exists a nonzero endomorphism  $u$  of  $M$  such that  $u(N) = 0$ ). A ring  $R$  is said to be retractable (resp co-retractable) if every left  $R$ -module is retractable (resp every left  $R$ -module is co-retractable). We characterize the co-retractable and retractable ring, the retractability of simple extension ring is also studied.

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## Indices of Rubin-Stark units

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ABSTRACT. Let  $K/k$  be a finite abelian extension of totally real number fields with Galois group  $G := \text{Gal}(K/k)$ .

We construct a  $\mathbb{Z}[G]$ -module generated by the Rubin-Stark elements associated with the extension  $K/k$ , and we show that this module is contained in the part of the exterior product of the module of  $S$ -units generated by a certain idempotent.

We study, then, the finitude of the obtained quotient and we evaluate it in terms of Sinnott's index and the class number. We analyse, ultimately, the behavior of this index in the cyclotomic  $\mathbb{Z}_p$ -tower.

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## Iwasawa theory and modular forms

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ABSTRACT. Iwasawa theory and modular forms are the most popular fields of number theory. We present a short talk on both subjects and relations with Main conjecture.

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## Euclidean Lattice and cryptography

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ABSTRACT. Cryptography allows the secret exchange of information on the Internet. The RSA method, although widely used, shows weaknesses and failures that remain to be solved. Cryptography on Lattice, a newly emerged concept of using operations that act on vectors and polynomials. This method based on the principle of "mixing", "combination" between message and public key P and disentangling with private key S, although promising, is still imperfect because the subspace generated by P is a subset generated by S. Based on the polynomial language "modulo  $x^4 - 1$ " and vectorial (geometric space); It uses the rotation operations of polynomials with coefficients that correspond to columns of the Cryptris. In this language, multiplication by X shifts the coefficients one notch to the left while the division shifts them to the right; While retaining the possibility of exchanging the division and multiplication operation.

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## On power serieswise Armendariz rings

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Dedicated to Our Professor El Amin KAIDI

ABSTRACT. In this paper, we investigate the transfer of the property of power serieswise Armendariz to trivial ring extensions, direct product of rings and the homomorphic image. The article includes a brief discussion of the scope and precision of our results.

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## When every pure ideal is projective

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ABSTRACT. In this Talk, we study the class of rings in which every pure ideal is projective. We investigate the stability of this property under homomorphic image, and its transfer to various contexts of constructions such as pullbacks, trivial ring extensions and amalgamation of rings.

Our results generate examples which enrich the current literature with new and original families of rings that satisfy this property.

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## Increasing the capacity of O-MIMO systems using MGDM technique

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(With O. EL Outassi and Y. Zouine)

**ABSTRACT.** The large bandwidth of multimode fiber (MMF), more particular graded index multimode fiber (GI-MMF) makes it a very attractive medium for multiservice transmission in local area networks. MGDM (Group Mode Diversity Multiplexing) is a multiplexing technique, which aims to improve the multimode optical fiber's performance by spatially multiplexing the data streams to be transmitted. We study in this work the optical MIMO systems (Multi-input Multi-output) over optical fiber on an MMF, specifically adapting the architecture of MIMO transmission systems. In this context we studied the technique of multiplexing Diversity modes group (MDGM) to assess transmission capacity. Indeed, the latter depends on the injection conditions and the state of the optical fiber.

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Mathematics Subject Classification (2010):.

Key words:MIMO optical; O-MGDM; multimode fiber; and transmission capacity.

## Fuzzy subgroup of an additive Fuzzy group

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ABSTRACT. In this abstract we investigated the fuzzy subgroups of a fuzzy group  $(F(\mathbb{R}^n), \tilde{+})$ , where  $\tilde{+}$  is the extension of  $+$  defined on  $\mathbb{R}^n$ , and we studied the relationship between the "crisp" subgroup of  $(\mathbb{R}^n, +)$  and the above fuzzy subgroups.

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**The cosine-sine functional equation on a semigroup with an  
involutive automorphism**

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ABSTRACT. By using algebraic methods, we determine the complex-valued solutions of the following extension of the Cosine-Sine functional equation

$$f(x\sigma(y)) = f(x)g(y) + g(x)f(y) + h(x)h(y), x, y \in S,$$

where  $S$  is a semigroup generated by its squares and  $\sigma$  is an involutive automorphism of  $S$ . We express the solutions in terms of multiplicative and additive functions.

## On nice modules and its dualization

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ABSTRACT. A ring with identity is called a clean ring if every element of the ring decomposes as a sum of an idempotent and a unit of the ring. Moreover, Khaksari and Moghimi [4] have defined a clean module, i.e. an  $R$ -module  $M$  is called a clean module if the ring  $S = \text{End}_R(M)$  of all endomorphisms of  $M$  is a clean ring. The important fact is any continuous module is a clean module (see Camillo et al. [1] and [2]). However, a submodule of a clean module does not have to be a clean module. For example the set of integer  $\mathbb{Z}$  is a not a clean module, meanwhile it is contained in a clean module  $\mathbb{Q}$  as  $\mathbb{Z}$ -modules. In their paper, Ismarwati et al. [3] proposed a module whose submodules are clean and called it a nice module. An interesting question is which modules are nice modules. We present some sufficient conditions under which a module is a nice module. Furthermore, a dualization of nice modules come from the observation of cleanness of factor modules of a module. A module  $M$  is called a co-nice module if all its factor module  $M/K$  is a clean module for any submodule  $K$  of  $M$ . We give some examples and sufficient conditions under which a module is a co-nice module.

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## **Emulate the neural network of nano arduino**

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(With R. Skouri and A. EL Abbassi)

**ABSTRACT.** This work presents an artificial neural network implementation in Arduino Board, simulated Network with Proteus ISIS.

The network described here is a feed-forward Backpropagation Network. It is considered as a best general purpose network for either supervised or unsupervised learning.

The code for the project is provided as Mplab . It is a plug and run for generating ".hex" file and uploading it to arduino in simulator. run the programme and there is a section of configuration information that can be used to quickly build and train a customized network.

The write-up provided here gives an overview of artificial neural networks, details of the sketch, it's an introduction to some of the basic concepts employed in feed forward networks and the backpropagation algorithm.

## On some conjectures on some sorts of Jordan derivations

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(With D. Bennis)

ABSTRACT. In this talk, we present a recent investigation on some conjectures related to some sorts of Jordan derivations.

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## On the determination of periods of linear recurrences

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(With O. Diankha, M. Mignotte and M. Sanghar)

ABSTRACT. The periodicity of LRS modulo  $p$ , with  $p$  a prime integer, was enough studied and it was particularly approached by L. Cerlienco, M. Mignotte and F. Piras.

O. Diankha provided results, in favour of sequences of the third degree, based on characteristic polynomial. We'll give a arithmetical interpretation of these results, which appear very simple to study the periodicity of LRS of the third degree.

We prove that the value of the period  $T_p$  of a linear recurring sequence modulo  $p$  is intimately linked to the decomposition of its companion polynomial modulo  $p$  and to deduce fast algorithms providing a multiple of  $T_p$ .

We study in detail the cost of the calculation of  $T_p$  for binary and cubic recurrences.

For the cubic recurrences, we give also the matrix method and will prove that the beginning of Berlekamp's algorithm can also lead to the same result.

We show that there are applications in shift register sequences.

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Mathematics Subject Classification (2010): .

Key words: Linear Recurring Sequences, period, modulo  $p$ , polynomials, Legendre symbols, companion polynomial, algorithm cost, Berlekamp's algorithm, rank of a matrix, law of reciprocity quadratic.

## Absolutely $lq$ -finite extensions

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ABSTRACT. Let  $K/k$  be purely inseparable extension of characteristic  $p > 0$ . By invariants, we characterize the measure of the size of  $K/k$ . In particular, we give a necessary and sufficient condition that  $K/k$  is of bounded size. Furthermore, in this note, we continue to be interested in the relationship that connects the restricted distribution of finitude at the local level of intermediate fields of a purely inseparable extension  $K/k$  to the absolute or global finitude of  $K/k$ . Part of this problem was treated successively by J.K Devney, and in my work with M. Chellali. The other part is the subject of this paper, it is a question of describing the absolutely  $lq$ -finite extensions. Among others, any absolutely  $lq$ -finite extension decomposes into  $w_0$ -generated extensions. However, we construct an example of extension of infinite size such that for any intermediate field  $L$  of  $K/k$ ,  $L$  is of finite size over  $k$ . In addition,  $K/k$  does not respect the distribution of horizontal finitude.

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Mathematics Subject Classification (2010): 12F15.

Key words: Purely inseparable, Relatively perfect, Degree of irrationality, Modular extension,  $q$ -finite extension,  $lq$ -finite extension, absolutely  $lq$ -finite extension.

## Classification of pairs of linear mappings between two vector spaces and between their quotient space and subspace

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ABSTRACT. The canonical form of matrices of pairs of linear mappings

$$\mathcal{A} : U \rightarrow V, \quad \mathcal{B} : U \rightarrow V$$

among finite dimensional vector spaces  $U$  and  $V$  over a field  $\mathbb{F}$  was given by Kronecker in 1890. The theory of such pairs, which is known as the *theory of matrix pencils*, is one of the most fruitful branches of linear algebra, with applications in many other areas. In this talk, we provide a canonical form of matrices of pairs of linear mappings

$$\mathcal{A}' : U/U' \rightarrow V', \quad \mathcal{B} : U \rightarrow V,$$

in which  $U, V$  are finite dimensional vector spaces over a field  $\mathbb{F}$ ,  $U' \subset U$  and  $V' \subset V$  are their subspaces, and  $U/U'$  is a quotient space. Related results were obtained earlier by Futorny and Sergeichuk.

This is a joint work with A. Dmytryshyn and T. Rybalkina.

## A construction and representation of some variable length codes

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**ABSTRACT.** *Let  $\Sigma$  be an alphabet. A subset  $X$  of the free monoid  $\Sigma^*$  is a code over  $\Sigma$  if for all  $m, n \geq 1$  and  $x_1, \dots, x_n, y_1, \dots, y_m \in X$ , the condition :*

*$x_1x_2\dots x_n = y_1y_2\dots y_m$  implies  $n = m$  and  $x_i = y_i$  for  $i = 1, \dots, n$ .*

*In other words, a set  $X$  is a code if any word in  $X^+$  can be written uniquely as a product of words in  $X$  [1]. It is not always easy to verify a given set of words is a code.*

*In this paper, we give the construction and representation by deterministic finite automata of some variable length codes.*

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Mathematics Subject Classification (2010): 94A60, 68Q42, 68Q70, 20M05.

Key words: Words and Languages, The free monoid and relatives, Morphism of monoids, deterministic finite automata.



## Skew cyclic codes over a principal ideal ring

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(With M. Ashraf )

ABSTRACT. Let  $\theta_t$  be an automorphism on ring  $R$ . Then a linear code  $C$  of length  $n$  over  $R$  is called a skew cyclic code or  $\theta_t$ -cyclic code if for each  $c = (c_0, c_1, \dots, c_{n-1}) \in C$  implies that

$$\sigma(c)(\theta_t(c_{n-1}), \theta_t(c_0), \dots, \theta_t(c_{n-2})) \in C.$$

In this paper, we study skew cyclic codes over the ring  $F_q + uF_q + vF_q$ , where  $u^2 = u$ ,  $v^2 = v$ ,  $uv = vu = 0$ ,  $q = p^m$  and  $p$  is a prime. We define a Gray map from  $F_q + uF_q + vF_q$  to  $F_q^3$  and investigate the structural properties of skew cyclic codes over  $F_q + uF_q + vF_q$  using decomposition method. It is shown that the Gray images of skew cyclic codes of length  $n$  over  $F_q + uF_q + vF_q$  are the skew 3-quasi cyclic codes of length  $3n$  over  $F_q$ . Finally, the idempotent generators of skew cyclic codes over  $F_q + uF_q + vF_q$  have also been discussed.

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Mathematics Subject Classification (2010): 94B05, 94B15.

Keywords: Gray map, Linear codes, Skew polynomial rings, Skew cyclic codes, Idempotent generators.

$L_d(1)$  is  $\mathcal{O}(\log\log\log d)$  for almost all square free  $d$

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**ABSTRACT.** Let  $d$  be a square free integer. Using Hardy-Ramanujan's value of normal order of  $\omega(d)$  we show that  $L_d(1) = \mathcal{O}(\log\log\log d)$  except on a negligible set. We note that the proof verifies Robin's inequality  $\sigma(n) < e^\gamma n \log\log n$  (equivalent form of Riemann Hypothesis) for such numbers.

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## The relationship between almost Dunford-Pettis operators and almost limited operators

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ABSTRACT. We investigate Banach lattices on which each positive almost Dunford-Pettis operator is almost limited and conversely.

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Mathematics Subject Classification (2010): 46B42, 47B60, 47B65.

Key words: Almost Dunford-Pettis operator, almost limited operator, the positive dual Schur property, order continuous norm, property (d).

## Family of functional inequalities for the uniform measure

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ABSTRACT. We consider the semigroup  $(P_t)_{t \geq 0}$  generated by the operator  $L := (1 - x^2) \frac{d^2}{dx^2} - 2x \frac{d}{dx}$ , on the interval  $[-1, 1]$  equipped with the probability measure  $\mu(dx) := \frac{dx}{2}$ . We establish, via a method involving probabilistic techniques, a family of inequalities which interpolate between the Sobolev and Poincaré inequalities.

## On the class group of formal power series rings

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(With S. Hizem)

ABSTRACT. Let  $\text{Cl}_t(A)$  denote the  $t$ -class group of an integral domain  $A$ . P. Samuel has established that if  $A$  is a Krull domain then the mapping  $\text{Cl}_t(A) \rightarrow \text{Cl}_t(A[[X]])$ , is injective and if  $A$  is a regular UFD then  $\text{Cl}_t(A) \rightarrow \text{Cl}_t(A[[X]])$ , is bijective. Later, L. Claborn extended this result in case  $A$  is a regular Noetherian domain. In this work we prove that the mapping  $\text{Cl}_t(A) \rightarrow \text{Cl}_t(A[[X]])$ ;  $[I] \mapsto [(I.A[[X]])_t]$  is an injective homomorphism and in case of an integral domain  $A$  such that each  $v$ -invertible  $v$ -ideal of  $A$  has  $v$ -finite type, we give an equivalent condition for  $\text{Cl}_t(A) \rightarrow \text{Cl}_t(A[[X]])$ , to be bijective, thus generalizing the result of Claborn.

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## Zero-divisor graphs of power series rings

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ABSTRACT. Let  $R$  be a commutative ring with identity,  $Z(R)$  its set of zero-divisors and  $N(R)$  its nilradical. The zero-divisor graph of  $R$  denoted by  $\Gamma(R)$ , is the graph with vertices  $Z(R) \setminus (0)$ , with distinct vertices  $x$  and  $y$  adjacent if and only if  $xy = 0$ . In this paper we give some results about a zero-divisors in the power series ring  $R[[X]]$ , and we study the diameter of  $\Gamma(R[[X]])$  in the case when  $N(R) = Z(R)$ . Also we give some results when  $N(R) \subsetneq Z(R)$ , among these case, we prove that  $\text{diam}(\Gamma(R)) = \text{diam}(\Gamma(R[X])) = 2$  and  $\text{diam}(\Gamma(R[[X]]) = 3$  if  $R$  is one of the following rings, *divided ring*, *PVR ring*, *chained ring* or  $R$  is a ring such that  $Z(R) = aR + I$  with  $a \in Z(R) \setminus N(R)$  and  $I \subsetneq (0 : a)$ .

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## Additive mappings on a prime rings with involution

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ABSTRACT. This talk deals with the investigation of the relationship between the structure of a ring  $R$  and the behaviour of some additive mappings defined on  $R$ . More precisely, we will consider functions satisfying some specific differential identities.

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## Counting twin primes

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ABSTRACT. In this talk we give two new formulae which count exactly the quantity of twin primes not greater than a certain given value  $36n^2 + 60n + 21$  and  $p_n^2 - 3$ . We use in these formulae the arithmetic progressions and the cardinality. In the first formula, we do not need to make any "primality" test and in the second formula we use the  $n$ -th prime number and we show the relation between counting primes and twin primes. We would also say that we have produced new algorithms to make such count.

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## Trivial Extensions defined by 2-absorbing-like conditions

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(With N. Mahdou)

Dedicated to Our Professor El Amin KAIDI

ABSTRACT. Let  $R$  be a commutative ring with  $1 \neq 0$ . The notion of 2-absorbing ideal and 2-absorbing primary ideal are introduced by Ayman BADAWI as a generalization of prime ideal and primary ideal respectively. A proper ideal  $I$  of  $R$  is called 2-absorbing ideal (resp., 2-absorbing primary ideal) if whenever  $a, b, c \in R$  with  $abc \in I$ , then  $ab \in I$  or  $ac \in I$  or  $bc \in I$  (resp.,  $ab \in \sqrt{I}$  or  $ac \in \sqrt{I}$  or  $bc \in \sqrt{I}$ ).

In this paper, we investigate the transfer of 2-absorbing-like properties to trivial ring extensions.

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## Bhargava rings over subsets

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(With I. Alrasasi)

ABSTRACT. Let  $D$  be an integral domain with quotient field  $K$  and let  $E$  be any nonempty subset of  $K$ . The Bhargava ring over  $E$  at  $x \in D$  is defined by  $\mathbb{B}_x(E, D) := \{f \in K[X] \mid f(xX + e) \in D[X], \forall e \in E\}$ . This ring is a subring of the ring of integer-valued polynomials over  $E$ . This paper studies  $\mathbb{B}_x(E, D)$  for an arbitrary domain  $D$ . we provide information about its localizations and transfer properties, describe its prime ideal structure, and calculate its Krull and valuative dimensions.

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Mathematics Subject Classification (2010): 13F20, 13B25, 13C15, 13B30.

Key words: Integer-valued polynomial, Bhargava ring, Prime ideal, Localization Residue field, Krull dimension, Valuative dimension.

## Counting the number of Fuzzy topologies

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**ABSTRACT.** The number of topologies defined on a finite set is an outstanding and open problem. There is no known explicit formula for the total number of topologies  $T(n)$ , one can define on an  $n$ -element set. The sequence  $T(n)$  is known just for  $n \leq 18$ , see [7].

Another approach of the subject is the enumeration according to the number of open sets. Let  $T(n, k)$  be the number of topologies on an  $n$ -element set having  $k$  ( $2 \leq k \leq 2^n$ ) open sets. The sequence  $T(n, k)$  is known just for  $k \leq 17$ , see [7]. It is also known that  $T(n, k) = 0$ , for  $3 \cdot 2^{n-2} < k < 2^n$  and  $n \geq 3$ .

On the other hand, fuzzy topological spaces satisfying some finiteness conditions have not been considered yet unlike the classical topology where this field is still active and attracting several researchers by its importance and by the numerous long-standing unsolved problems, [1], [2]. The main purpose of this work is to remedy this lack and initiate the corresponding fuzzy side of these problems.

For a set  $X$  of cardinality  $n$ , and a complete lattice  $M$  of cardinality  $m$ , let  $T_{\mathcal{F}}(n, m)$  be the total number of fuzzy topologies on  $X$ , with membership values in  $M$ , and let  $T_{\mathcal{F}}(n, m, k)$  be the number of fuzzy topologies on  $X$ , with membership values in  $M$ , having  $k$  open sets. We have trivially  $T_{\mathcal{F}}(n, m, 2) = 1$ ,  $T_{\mathcal{F}}(n, m, 3) = m^n - 2$ . For  $k \geq 4$ , calculations are not as immediate as for  $k = 2$ , or 3.

In this work we give conditions under which the number of fuzzy topologies on  $\mathcal{F}$  is finite. Then we compute  $T_{\mathcal{F}}(n, m, 4)$  and  $T_{\mathcal{F}}(n, m, 5)$ . Then non-discrete topologies of maximal cardinalities are investigated, where the number and the cardinality of such topologies is established. Several known results for finite classical topologies are obtained as corollaries of the established results in this work for the fuzzy setting. We conclude with some open questions and directions for other investigations.

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## Additivity of Jordan higher derivable maps on alternative rings

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ABSTRACT. Let  $\mathcal{R}$  be an alternative ring (not necessarily have identity element). A map (not necessarily additive)  $d : \mathcal{R} \rightarrow \mathcal{R}$  is said to be a Jordan (resp. Jordan triple) derivable map if  $d(xy+yx) = d(x)y+xd(y)+d(y)x+yd(x)$  (resp.  $d(xyx) = d(x)yx + xd(y)x + xyd(x)$ ) holds for all  $x, y \in \mathcal{R}$ . In the present paper, it is shown that every Jordan (triple) derivable map is additive under certain assumptions.

## **$p$ -Local formations whose length $\leq 3$**

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ABSTRACT. Throughout this paper, all groups are finite. Recall that a formation is a homomorphic  $\mathfrak{F}$  of groups such that each group  $G$  has a smallest normal subgroup (denoted by  $G^{\mathfrak{F}}$ ) whose quotient is still in  $\mathfrak{F}$ . A formation  $\mathfrak{F}$  is said to be  $p$ -local if it contains each group  $G$  with  $G/(\Phi(G) \cap O_p(G)) \in \mathfrak{F}$ . Our main goal here is to prove the following theorem

Theorem: A non-reducible  $p$ -local formation  $\mathfrak{F}$  has length 3 if and only if  $\mathfrak{F} = Lform_p(G)$  such that the one of the following conditions is satisfied:

- (1)  $R$  is a non-abelian  $pd$ -group and  $G/R$  is  $p$ -group.
- (2)  $G$  is cyclic group of order  $q^2$ , where  $q \neq p$ .
- (3)  $G$  is non-abelian group of order  $q^3$ , where  $q \neq p$ .
- (4)  $G$  is  $p'$ -group,  $R \subseteq \phi(G)$  and  $G/R \cong A \times A \times A \cdots \times A$ , where  $A$  is simple group.

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## On the structure of 2-group $\text{Gal}(K_2^{(\infty)}/K)$ of some imaginary quartic number field $K$

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ABSTRACT. Let  $k$  be an algebraic number field. For a prime number  $p$ , let  $k_p^{(0)} = k$  and  $k_p^{(i)}$  denote the Hilbert  $p$ -class field of  $k_p^{(i-1)}$  for  $i \geq 1$ . Then we have the sequence of fields

$$k = k_p^{(0)} \subseteq k_p^{(1)} \subseteq \dots \subseteq k_p^{(\infty)} = \bigcup_{i=0}^{\infty} k_p^{(i)},$$

that is called the  $p$ -class field tower of  $k$ . Let  $K$  be an imaginary quartic cyclic number field of type  $(2, 2, 2)$ , i.e. its 2-class group  $\mathbf{C}_{K,2} \simeq \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ , under some conditions. Then, in this work, we determine the structure of 2-group  $\text{Gal}(K_2^{(\infty)}/K)$ .

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## **Algebraic independence and algebraic independence measure of real numbers**

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**ABSTRACT.** In this paper we provide some properties and results of results on transcendental and algebraic independence of real numbers. Then we give sufficient conditions on the real numbers  $A_1, A_2, \dots, A_k$ , where  $k \geq 3$  which ensure the algebraic independence of these numbers. The used method also permits us to calculate an algebraic independence measure of the above numbers.

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Mathematics Subject Classification (2010): .

Key words: Continued fraction, Transcendence, algebraic independence, measure.

## Property $(UW_{\Pi})$ under perturbations

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ABSTRACT. Property  $(UW_{\Pi})$  for a bounded linear operator  $T \in L(X)$  on a Banach space  $X$  means that the points  $\lambda$  of the approximate point spectrum for which  $\lambda I - T$  is upper semi-Weyl are exactly the spectral points  $\lambda$  such that  $\lambda I - T$  is Drazin invertible. In this work we study the stability of this property under some commuting perturbation, as quasi-nilpotent perturbation and more in general, under Riesz commuting perturbations. We also study the transmission of property  $(UW_{\Pi})$  from  $T$  to  $f(T)$ , where  $f$  is an analytic function defined on a neighborhood of the spectrum of  $T$ .

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## Parametrizing MED semigroups with multiplicity up to five

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ABSTRACT. In a recent article to appear (Parametrizing Arf numerical semigroups, Journal of Algebra and its Applications, Vol. 16, No. 11(2017), DOI: 10.1142/S02194988177502097), P. A. GarciaSanchez, B. A. Hredia, J. C. Rosales and the author gave parametrization of Arf numerical semigroups with multiplicity up to seven and given conductor. In the present work, a new characterization of MED semigroups (numerical semigroups having maximal embedding dimension) is given and this characterization is used to parametrize MED semigroups with multiplicity up to five and given conductor.

## A note on finite products of fields

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ABSTRACT. Let  $R$  be a commutative ring. Suppose that  $R$  is zero-dimensional, it would be interesting to check whether  $R$  contains a finite product of fields. Recently many papers have studied Artinian subrings of a commutative ring and direct limit of finite product of fields ([4, 5, 6, 8]). Recall that Artinian rings form an important class of zero-dimensional rings. Moreover, an Artinian ring has only finitely many idempotent elements. Essentially, the characterization of the set of Artinian subrings of a commutative ring is known (see [6]). In this talk we are interested in the Artinian overring of pair of rings, that means, we are looking for intermediate Artinian rings between  $R$  and  $T$ , where  $R$  is a subring of a von Neumann regular ring  $T$ .

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## Left and right spectra of operator matrices

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ABSTRACT. In this paper, we investigate the limit points set of left and right spectra of upper triangular operator matrices  $M_C = \begin{pmatrix} A & C \\ 0 & B \end{pmatrix}$ . We prove that  $acc(\sigma_*(M_C)) \cup W_{acc\sigma_*} = acc(\sigma_*(A)) \cup acc(\sigma_*(B))$  where  $W_{acc\sigma_*}$  is the union of certain holes in  $acc(\sigma_*(M_C))$ , which happen to be subsets of  $acc(\sigma_l(B)) \cap acc(\sigma_r(A))$  and  $\sigma_*(\cdot)$  can be equal to the left or right spectrum. Furthermore, several sufficient conditions for  $acc(\sigma_*(M_C)) = acc(\sigma_*(A)) \cup acc(\sigma_*(B))$  holds for every  $C \in \mathcal{B}(Y, X)$  are given.

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## The discrete logarithm problem modulo odd integers

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ABSTRACT. Let  $m$  be a fixed odd positive integer. We define the recurrent sequence  $(v_n)_{n \in \mathbb{N}}$  by the initial term  $v_0 \in \mathbb{N}$  with  $0 < v_0 < p$ , and the relations:

$$v_{n+1} = \begin{cases} \frac{v_n}{2} & \text{if } v_n \text{ is even} \\ \frac{m + v_n}{2} & \text{otherwise.} \end{cases}$$

In this communication, we try to generalize a previous work done with the parameter  $m$  as a prime number. It will be shown that the sequence  $(v_n)_{n \in \mathbb{N}}$  is useful when solving the modular equation  $a^x \equiv b [m]$  in the multiplicative group  $((\frac{\mathbb{Z}}{m\mathbb{Z}})^*, \cdot)$ . We also analyze the case when  $m$  is the power of a prime number.

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## Power series over strongly Hopfian bounded rings

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ABSTRACT. An  $R$ -module  $M$  is called strongly Hopfian (respectively bounded) if for every endomorphism  $f$  of  $M$  the chain  $\text{Ker}(f) \subseteq \text{Ker}(f^2) \subseteq \dots$  stabilizes (respectively there exists a positive integer  $n$  such that for every endomorphism  $f$  of  $M$ ,  $\text{Ker}(f^n) = \text{Ker}(f^{n+1}) = \dots$ ). The ring  $R$  is strongly Hopfian (respectively bounded) if it so as an  $R$ -module. Let  $R$  be a commutative unitary ring. We show that  $R[[X]]$  is strongly Hopfian bounded if and only if  $R$  has a strongly Hopfian bounded extension  $T$  such that  $I_c(T)$  contains a regular element of  $T$ . We deduce that if  $R[[X]]$  is strongly Hopfian bounded, then so is  $R[[X, Y]]$  where  $X, Y$  are two indeterminates over  $R$ . Also we show that if  $R$  is embeddable in a zero-dimensional strongly Hopfian bounded ring, then so is  $R[[X]]$  (this generalizes most results of S. Hizem). For a chained ring  $R$ , we show that  $R[[X]]$  is strongly Hopfian if and only if  $R$  is strongly Hopfian.

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Mathematics Subject Classification (2010): 13A15 - 13F25 - 13F30.

Key words: embeddability in a zero-dimensional ring, strongly Hopfian (bounded) ring, power series ring, SFT-ring, chained ring.

## Algebraic Schur complement approach for a finite volume discretization of a non linear 2d convection diffusion equation

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ABSTRACT. This work deals with a domain decomposition approach for a non-linear convection diffusion equation. The domain of calculation is decomposed into  $q \geq 2$  non-overlapping sub-domains. On each sub-domain the linear part of the equation is discretized using implicit finite volumes scheme (FV) and the non linear convection term is integrated explicitly into the scheme. As non-overlapping domain decomposition, we propose the Schur Complement (SC) Method. The proposed approach is applied for solving the local boundary sub-problems. The numerical experiments applied to Burgers equation show the interest of the method compared to the global calculation. The proposed algorithm has both the properties of stability and efficiency. It can be applied to more general nonlinear PDEs and can be adapted to different FV schemes.

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## Some commutativity results for prime near- ring involving derivations

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ABSTRACT. In this talk we investigate some commutativity results for a prime near-ring as follows: Let  $N$  be a zero-symmetric left prime near-ring and  $\delta : N \rightarrow N$  a derivation such that (i)  $\delta([a, b]) = [a, \delta(b)]$ ; (ii)  $[a, \delta(b)] = [a, b]$ ; (iii)  $[a, \delta(b)] \in Z(N)$ ; (iv)  $\circ\delta(b) \in Z(N)$ ; and (v)  $a \circ \delta(b) = b \circ a \forall a, b \in N$ . This paper first establish the commutativity of prime near-rings satisfying one of the above conditions [(i) - (v)] that associates with a derivation  $\gamma$  on  $N$ . Secondly, it is shown that  $N$  is a commutative ring if a prime near-ring  $N$  admits a derivation  $\delta$  with a semi group ideal of  $N$  involving (i), (ii), and other related identities. In addition, examples are given to validate the assumptions in the hypothesis, which are not superfluous. Finally, we close our discussion with some open problems

**Pure ideals in ordered  $\Gamma$ -semigroups and right regular weakly ordered  $\Gamma$ -semigroups**

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**ABSTRACT.** We introduce the notions of pure ideals, left(right) weakly pure ideals and purely prime ideals in an ordered  $\Gamma$ -semigroup and study some of their properties. We, then, define a right weakly regular ordered  $\Gamma$ -semigroup and study various interplays between ideals, bi-ideals and interior ideals within this ordered semigroup. Finally, we characterize right weakly regular ordered  $\Gamma$ -semigroups through ideals, bi-ideals and interior ideals of an ordered  $\Gamma$ -semigroup.



## Gorenstein injective modules with respect to a semidualizing bi-module

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ABSTRACT. Let  ${}_S V_R$  a semidualizing  $(S-R)$ -bimodule over the associative rings  $R$  and  $S$  and let  $\mathcal{I}_V(R) := \{\text{Hom}_S(V, I) : I \text{ is an injective } S\text{-module}\}$ . By a  $V$ -Gorenstein injective module we mean an  $R$ -module,  $N$  say, possessing a  $\text{Hom}_R(\mathcal{I}_V(R), -)$  and  $\text{Hom}_R(-, \mathcal{I}_V(R))$  exact exact complex  $\cdots \rightarrow I_1 \xrightarrow{d_0} I_0 \rightarrow I^0 \xrightarrow{d^0} I^1 \rightarrow \cdots$  such that  $I_i, I^i \in \mathcal{I}_V(R)$  and  $N \cong \text{Im}(I_0 \rightarrow I^0)$ . Some homological properties of the class of  $V$ -Gorenstein injective modules and its connection with the Auslander class  $\mathcal{A}_V(R)$  and the class of strongly  $V$ -Gorenstein injective modules is investigated. Also, a characterization of finiteness of  $V$ -Gorenstein injective dimension in terms of vanishing of the relative cohomological functor  $\text{Ext}_{\mathcal{I}_V(R)}^{\geq 0}(-, N)$  is given.

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**On some steintiz properties on finitely generated submodules of  
free modules**

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ABSTRACT. Let  $R$  be a commutative ring and let  $M$  be an  $R$ -module. Following M. Lazarus [1], we say that  $M$  satisfies property  $(P)$  if any two maximal linearly independent subsets of  $M$  have the same cardinality. In this talk we give conditions on  $R$  under which any finitely generated submodule of a free  $R$ -module satisfies property  $(P)$ .

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## Généralisation d'une congruence d'Emma Lehmer

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RÉSUMÉ. L'étude de la somme  $\sum_{r=1}^{p-1} r^{2k}$ , modulo un nombre premier impair  $p$  a fait et continue de faire l'objet de nombreux travaux . En 1938, Emma Lehmer obtenait pour cette somme une congruence modulo  $p^3$  faisant intervenir les nombres et polynomes de Bernoulli .Dans cette communication, nous développons une nouvelle approche nous permettant d'améliorer la congruence d'Emma Lehmer en une congruence modulo  $p^s$ , avec  $s \in \{4, 5\}$ .

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## Nonlinear commutant preservers

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(With H. Benbouziane, Y. Bouramdane, and M. E. Kettani)

ABSTRACT. Let  $\mathfrak{B}(X)$  be the algebra of all bounded linear operators on Banach space  $X$ . We determine the form of maps (not necessarily linear)  $\phi : \mathfrak{B}(X) \rightarrow \mathfrak{B}(X)$  which satisfying the following condition of preservation  $\{\phi(A) \diamond \phi(B)\}' = \{A \diamond B\}'$  for different kinds of binary operations  $\diamond$  on operators such as the product  $AB$ , triple product  $ABA$ , and Jordan product  $AB+BA$  for all  $A, B \in \mathfrak{B}(X)$  where  $\{A\}'$  is the set of operators commuting with  $A \in \mathfrak{B}(X)$ .

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## Zero-divisor graphs in commutative rings

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ABSTRACT. In this talk, we present the recent and active area of zero-divisor graphs of commutative rings. Notable algebraic and graphical results are mentioned.

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## A computation in temperley-Lieb algebra

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(With S. Kim)

ABSTRACT. For Hecke algebras and Temperley-Lieb algebras of type  $A$  as well as for Ariki-Koike algebras, their Gröbner-Shirshov bases were constructed in [2, 3]. In this note, we deal with Temperley-Lieb algebras of type  $B$ , extending the result in [2, §6]. By completing the relations coming from a presentation of the Temperley-Lieb algebra, we find its Gröbner-Shirshov basis to obtain the corresponding set of standard monomials. The explicit multiplication table between the monomials follows naturally.

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Mathematics Subject Classification (2010): Primary 20F55, Secondary 05E15, 16Z05.

Key words: Temperley-Lieb algebra, Gröbner-Shirshov basis

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## Ad-nilpotent elements of semiprime rings with involution

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ABSTRACT. Let  $R$  be an  $n!$ -torsion free semiprime ring with involution  $*$  and with extended centroid  $C$ , where  $n > 1$  is a positive integer. We characterize  $a \in K$ , the Lie algebra of skew elements in  $R$ , satisfying  $(\text{ad}_a)^n = 0$  on  $K$ . This generalizes both Martindale and Miers' theorem [?] and the theorem of Brox et al. [?]. To prove it we first prove that if  $a, b \in R$  satisfy  $(\text{ad}_a)^n = \text{ad}_b$  on  $R$ , where either  $n$  is even or  $b = 0$ , then  $(a - \lambda)^{\lfloor \frac{n+1}{2} \rfloor} = 0$  for some  $\lambda \in C$ .

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Mathematics Subject Classification (2010): 16N60, 16W10, 17B60.

Key words: Semiprime ring, Lie algebra, Jordan algebra, faithful  $f$ -free, involution, skew (symmetric) element, ad-nilpotent element, Jordan element.

## Global dimension of bi-amalgamated algebras

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ABSTRACT. Let  $f : A \rightarrow B$  and  $g : A \rightarrow C$  be two ring homomorphisms and let  $J$  and  $J'$  be two ideals of  $B$  and  $C$ , respectively, such that  $f^{-1}(J) = g^{-1}(J')$ . The bi-amalgamation of  $A$  with  $(B, C)$  along  $(J, J')$  with respect to  $(f, g)$  is the subring of  $B \times C$  given by

$$A \bowtie^{f,g} (J, J') = \{(f(a) + j, g(a) + j') \mid a \in A, (j, j') \in J \times J'\}$$

The aim of this paper is to characterize the global dimension of bi-amalgamated algebras.

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## On $(\sigma, \delta)$ -skew McCoy modules

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ABSTRACT. Let  $(\sigma, \delta)$  be a quasi derivation of a ring  $R$  and  $M_R$  a right  $R$ -module. In this work, we introduce the notion of  $(\sigma, \delta)$ -skew McCoy modules which extends the notion of McCoy modules and  $\sigma$ -skew McCoy modules. This concept can be regarded also as a generalization of  $(\sigma, \delta)$ -skew Armendariz modules. Some properties of this concept are established and some connections between  $(\sigma, \delta)$ -skew McCoyness and  $(\sigma, \delta)$ -compatible reduced modules are examined. Also, we study the property  $(\sigma, \delta)$ -skew McCoy of some skew triangular matrix extensions  $V_n(M, \sigma)$ , for any nonnegative integer  $n \geq 2$ . As a consequence, we obtain: (1)  $M_R$  is  $(\sigma, \delta)$ -skew McCoy if and only if  $M[x]/M[x](x^n)$  is  $(\bar{\sigma}, \bar{\delta})$ -skew McCoy, and (2)  $M_R$  is  $\sigma$ -skew McCoy if and only if  $M[x; \sigma]/M[x; \sigma](x^n)$  is  $\bar{\sigma}$ -skew McCoy.

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Mathematics Subject Classification (2010): 16S36, 16U80.

Key words: McCoy module,  $(\sigma, \delta)$ -skew McCoy module, semicommutative module, Armendariz module,  $(\sigma, \delta)$ -skew Armendariz module, reduced module.

## **$S$ -Prime ideals over $S$ -Noetherian ring**

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ABSTRACT. Let  $A$  be a commutative ring with identity and  $S \subseteq A$  a multiplicative subset. In this paper we introduce the concept of  $S$ -prime ideal which is a generalization of prime ideal. Let  $I$  be an ideal of  $A$  disjoint with  $S$ . We say that  $I$  is an  $S$ -prime ideal if there exists  $s \in S$  such that for all  $a, b \in A$  with  $ab \in I$  then  $sa \in I$  or  $sb \in I$ . In this work we show that  $S$ -prime ideals enjoy analogs of many properties of prime ideals and we study then over  $S$ -Noetherian rings.

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## **Domains with invertible-radical factorization**

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**ABSTRACT.** In this paper we study those integral domains in which every proper ideal can be written as an invertible ideal multiplied by a nonempty product of proper radical ideals.

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Mathematics Subject Classification (2010): 16W25; 46J10.

Key words: Prime ring; Banach algebra; Derivations; generalized derivation.

## A new framework for Zakat calculation using mathematics equations based on XML technology

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**ABSTRACT.** Zakat is one of the five pillars of Islam, and is expected to be paid by all practicing Muslims who have the financial means. This paper suggests an advanced Islamic framework for the purpose of calculating the amount owed to pay using mathematics equation. The proposed framework can be used in two ways. Firstly, the full Zakat calculator for calculating amount of all Zakatable wealth such as cash in hand, business, silver and gold. Secondly, the partial Zakat calculator for calculating amount owed to pay of a particular wealth such as gold, precious stones, silver, landed property, and more. Moreover, it allows calculating the Zakat with the easy, accurate and elaborate way and getting step by step instructions on how to accurately calculate the amount of Zakat. Furthermore, it helps to understand and handle the Zakatable wealth in the most effective and efficient way. This paper also presents some cases study scenarios emphasizing the benefits enabled by the proposed framework. Experimental results show that, compared with traditional solutions, our framework is more scalable and high-performance.

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Mathematics Subject Classification (2010): 16E50, 16W25, 16N60, 16W99.

Key words: Calculator; Zakat; Mathematics equations; Framework; XML; MDA; MOF.

## Another Gorenstein analogue of flat modules

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ABSTRACT. In this communication, we consider the notion of algebraic zero-divisors in special algebras that are, the hermitian Banach algebras. Let  $A_+^0 = \{x \in A : x = x^* \text{ and } Sp(x) \subset \mathbb{R}^{+*}\}$  and  $A_+ = \{x \in A : x = x^* \text{ and } Sp(x) \subset \mathbb{R}^+\}$  the convex cone of positive elements of a hermitian Banach algebra  $A$ . We show that if  $A_+$  does not contain zero divisors then,  $Sym(A) \cap G$  is the disjoint union of  $A_+^0$  and  $(-A_+^0)$ . In other words a invertible and self-adjoint element is either strictly positive or strictly negative. We remark that, in a hermitian Banach algebra without zero-divisors, any self-adjoint element whose spectrum contains both negatif and positif real numbers is necessarily not invertible. So the existence of such elements informs us for the existence of zero-divisors. As a corollary we obtain that the spectrum of each self-adjoint element will be an interval when  $A_+$  contains no zero divisors. Finally we show that a hermitian Banach algebra containing no algebraic zero divisors and for which every positive element has a positive square root is isomorphic to  $\mathbb{C} + Rad(A)$  (The spectrum of each element contains a single complex number)

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Mathematics Subject Classification (2010): 16D40, 16E30,16E65.

Key words: Gorenstein projective module, Gorenstein flat module, glat module, coherent ring, preenvelope, precover.

## Generalization of direct injective modules

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ABSTRACT. In this paper we generalize the concept of direct injective (or  $C_2$ ) modules to finite direct injective modules. Some properties of finite direct injective modules are investigated. We show that direct summand of finite direct injective modules inherits the property, while direct sum need not. Some classes of rings are characterize with the help of finite direct injective modules.

## Stark units and Iwasawa theory

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ABSTRACT. In this talk, we will discuss the relationship between the characteristic ideal of the  $\chi$ -quotient of the projective limit of the ideal class groups to the  $\chi$ -quotient of the projective limit of units modulo Stark units, in the non semi-simple case, for some  $\overline{\mathbb{Q}_p}$ -irreducible characters  $\chi$ .

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## **Théorème de Gel'fand-Mazur-Kaplansky**

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RÉSUMÉ. Le théorème classique de Gel'fand-Mazur affirme que toute  $\mathbb{C}$ -algèbre associative normée de division est isomorphe à  $\mathbb{C}$ . Nous donnons une esquisse de sa démonstration puis prouvons le théorème de Gel'fand-Mazur-Kaplansky qui affirme que toute  $\mathbb{R}$ -algèbre associative normée de division est isomorphe à  $\mathbb{R}$ ,  $\mathbb{C}$  ou  $\mathbb{H}$  (l'algèbre réelle des quaternions de Hamilton).



## Approximation of a special function by the continued fractions

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(With A. Chillali and A. Kacha)

ABSTRACT. In this work we will introduce the notion of continued fractions, which plays a very important role in mathematical domains such as: the approximation of irrationals, the resolution of equations, the study of transcendental numbers, the fractional matrix calculus, etc. We will give an approximation of a special function using the theory of continued fractions.

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## Real pre-Hilbert algebras satisfying $\|x^2\| = \|x\|^2$

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ABSTRACT. Let  $A$  be a real (non-associative) algebra which is normed as real vector space, with a norm  $\|\cdot\|$  deriving from an inner product. In this talk we characterize the real pre-Hilbert commutative algebras without divisors of zero and containing a nonzero element  $a$  such that  $\|ax\| = \|a\|\|x\|$  and  $\|x^2\| = \|x\|^2$  for any  $x \in A$ . This generalizes a well-known theorem by Urbanik and Wright asserting that every real commutative absolute valued algebra is isomorphic to  $\mathbb{R}, \mathbb{C}$  or  $\mathbb{C}^*$  [6]. We also show that every real pre-Hilbert normed algebraic algebra of degree 2 and containing a nonzero central idempotent  $f$  such that  $\|f\| = 1$  is flexible and satisfying  $\|x^2\| = \|x\|^2$  for any  $x \in A$ . The assumptions algebraic algebra and of degree 2 are essential and the counters-examples are given in [2, 3]. Moreover, we classify the real pre-Hilbert algebras without divisors of zero and having dimension 2 such that  $\|x^2\| = \|x\|^2$  for any  $x \in A$ . This last generalizes previously known results of A. Rodríguez [5]. Finally, we prove that every real algebra, with unit element  $e$ , without divisors of zero, containing a nonzero central element  $a$  which is linearly independent to  $e$ , and algebraic of degree  $\neq 8$ , is isomorphic to  $\mathbb{C}$ . The latter completes the results done by O. Diankha et al [1] and generalizes our results given in [4].

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Mathematics Subject Classification (2010): .

Key words: Pre-Hilbert (commutative, flexible) algebra, division algebra, normed algebra, central element.

## Tri-additive maps and local generalized $(\alpha, \beta)$ -derivations

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ABSTRACT. Let  $R$  be a prime ring with nontrivial idempotents. We characterize a tri-additive map  $f: R^3 \rightarrow R$  such that  $f(x, y, z) = 0$  for all  $x, y, z \in R$  with  $xy = yz = 0$ . As an application, we show that in a prime ring with nontrivial idempotents, any local generalized  $(\alpha, \beta)$ -derivation or generalized Jordan triple  $(\alpha, \beta)$ -derivation is a generalized  $(\alpha, \beta)$ -derivation.

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## **A study of non-additive maps in $\Gamma$ -structure of rings and near-rings**

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ABSTRACT. The purpose of this article is to prove some results which are of independent interest and related to non-additive maps on  $\Gamma$ - structure of rings and near-rings. Further, examples are given to demonstrate that restrictions imposed on the hypothesis of several results are not superfluous.

**On centrally-extended multiplicative  
(generalized)- $(\alpha, \beta)$ -derivations in semiprime rings**

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**ABSTRACT.** Let  $R$  be a ring with center  $Z$  and  $\alpha, \beta$  and  $d$  mappings of  $R$ . A mapping  $F$  of  $R$  is called a centrally-extended multiplicative (generalized)- $(\alpha, \beta)$ -derivation associated with  $d$  if  $F(xy) - F(x)\alpha(y) - \beta(x)d(y) \in Z$  for all  $x, y \in R$ . The objective of the present paper is to study the following conditions: (i)  $F(xy) \pm \beta(x)G(y) \in Z$ , (ii)  $F(xy) \pm g(x)\alpha(y) \in Z$  and (iii)  $F(xy) \pm g(y)\alpha(x) \in Z$  for all  $x, y$  in some appropriate subsets of  $R$ , where  $G$  is a multiplicative (generalized)- $(\alpha, \beta)$ -derivation of  $R$  associated with the map  $g$  on  $R$ .

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Key words: Semiprime ring, left ideal, multiplicative (generalized)-derivation, multiplicative (generalized)- $(\alpha, \beta)$ -derivation, centrally-extended generalized  $(\alpha, \beta)$ -derivation, centrally-extended multiplicative (generalized)- $(\alpha, \beta)$ -derivation, generalized  $(\alpha, \beta)$ -derivation.

**Continued fraction expansions of the quasi-arithmetic  
power means of positive matrices with parameter  $(p, \alpha)$**

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ABSTRACT. The goal of this paper is to provide an efficient method for computing the quasi-arithmetic power means of two positive matrices with parameter  $(p, \alpha)$  by using the continued fractions with matrix arguments. Furthermore, we give some numerical examples which illustrated the theoretical results.

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## Factorization with respect to multiplicatively closed subsets of a ring which split a module

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ABSTRACT. Let  $R$  be a commutative ring,  $S$  a saturated multiplicatively closed subset (SMCS for short, sometimes called a divisor-closed multiplicative submonoid) of  $R$  (we let  $0 \in S$ , that is,  $S = R$ ) and  $M$  a unitary  $R$ -module. Also here  $Z(M)$  is the set of zero-divisors of  $M$ . Many have tried to find relations between factorization properties of  $R$  (such as unique, finite or bounded factorization) and those of  $S^{-1}R$ , for a SMCS  $S$  of  $R$  (see, for example, [1]). The aim of this research is to show how generalizing factorization properties to modules (see [2, 4]) and with respect to a SMCS ([3]), could be utilized to give a better insight on this problem.

In this talk, first we briefly review the notations and definitions of  $S$ -factorization in modules. Then using these notions we present the concept of SMCS's which split a module and use it to prove: "Suppose that  $S \subseteq S'$  are two SMCS's of  $R$  such that  $S$  splits  $M$  and  $S \cap Z(M) = S' \cap Z(M) = \emptyset$ . Let  $\mathbf{P} \in \{\text{présimplifiable, BFM, FFM, HFM, UFM}\}$ . Then  $M$  is  $S'$ - $\mathbf{P}$  if and only if  $M$  is  $S$ - $\mathbf{P}$  and  $S^{-1}M$  is  $S^{-1}S'$ - $\mathbf{P}$ ." In the case  $S' = R$ , this gives some Nagata type theorems on factorization properties of modules. Finally, by an example ( $R = A + xB[x]$  and  $S = A^* \cup \{bx^n | b \in B^*, n \in \mathbb{N}\}$ , where  $A \subseteq B$  are two domains) we show how our results could be applied in the case that  $M = S' = R$ , to study factorization properties of a domain. We give new and simpler proofs for previously known theorems which characterize when  $R$  in this example is a BF, FF, HF or UF domain.

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Key words: splitting multiplicatively closed subset; factorization; atomicity.

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## Containment of Fuzzy subgroups in a direct product of finite symmetric groups

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**ABSTRACT.** One of the most important problems of fuzzy group theory is to classify the fuzzy subgroups structure and count the number of all distinct fuzzy subgroups of a finite groups. This area of research has enjoyed a rapid development in the last few years. First, an equivalence relation on the set of all fuzzy subgroups of a group  $G$  is defined. Without any equivalence relation on fuzzy subgroups of group  $G$ , the number of fuzzy subgroups is infinite, even for the trivial group.

In this paper a first step in classifying the fuzzy subgroups structure of a finite symmetric groups  $S_n \times S_m$  ( $n = m, n = 2 \& m \geq 3$ ) is made. An explicit formula for the number of distinct fuzzy subgroups of  $S_n \times S_m$  is indicated. We also count the number of fuzzy subgroups for a particular class of finite symmetric groups. As a guiding principle in determining the number of these classes, note that an essential role in solving our counting problem is played again by the Inclusion-Exclusion Principle. It leads us to some recurrence relations, whose solutions have been easily found.



## A new smoothing method based on diffusion equation and K-means clustering

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**ABSTRACT.** When an image is acquired by a camera or other imaging system, the vision system for which it is intended is often unable to directly use it. The image may be corrupted because of random variations in intensity, variations in illumination, or poor contrast that must be dealt with in the early stages of vision processing. The main goal of this paper is to discuss partial differential equation methods for image enhancement aimed at eliminating these undesirable characteristics. We propose a new filtering method, based on diffusion equation and k-means segmentation. The experimental results show that the proposed method has a better smoothing performance compared with the Perona-Malik method that uses the anisotropic diffusion. In addition, the proposed approach is simple and can provide a better smoothing in a few iterations, which gives, in a short execution time, a better image filtering.

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## **Crypto système à clé publique de McEliece basé sur les produits codes matrices**

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**RÉSUMÉ.** Les applications des produits matrice codes ont également une utilisation pratique en théorie des codes, dans la création des produits matrice codes en cryptosystème PMC est un cryptosystème de McEliece basés sur les produits matrice codes PMC, une famille de codes de distance minimale. Dans cette communication, on essaye d'introduire un crypto système à clé publique utilisant des codes correcteurs d'erreurs (c'est un système deux en un). Le système étudié est le crypto système de McEliece utilisant le produit des codes et des matrices.

Nous allons dans cette communication : faire une cryptographie en proposant une amélioration de l'attaque de décodage, précisément celui du décodage par ensemble d'information sur les codes binaires et ternaires. Finalement on va proposer une application du cryptosystème de Mc.Eliece en proposant une utilisation des : «Matrix-Product Codes».

## On Parry invariant of a quintic cyclic field

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ABSTRACT. Let us suppose that  $K/Q$  is a quintic cyclic field such that the conductor of  $K$  is divisible exactly by two primes. The aim of this work is to give a table of Parry invariant of  $K$  in the case which  $25 \parallel h_K$  where  $h_K$  is the classes group of  $K$ .

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## Les groupes de coxeter et le problème de distance d'inversion

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RÉSUMÉ. Comme les processus d'inversions ne peuvent pas être observer directement, les modèles mathématiques associés sont nécessaires pour tirer des inférences à partir des données génomiques à propos des processus évolutifs. Chaque modèle peut être utiliser aussi pour établir une métrique (distance) associée à l'espace des arrangements génomiques. Alternativement, ces métriques peuvent servir à la reconstruction des arbres phylogéniques (évolution et développement des espèces vivantes).

Une approche d'un point de vue algébrique à fin d'en extraire des informations sur ses processus est de modéliser ses phénomènes par des modèles basés sur la théorie des groupes. De cette façon on pourra traduire des questions à propos de la distance d'inversion en des questions concernant les groupes et pour s'y faire on utilisera plus précisément les groupes de Coxeter.

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Mathematics Subject Classification (2010) : .

Key words : Systeme de Coxeter, groupe symetrique affine, problème de distance d'inversion, inversion paracentrique, gène, phylogénie.

## G-ring pairs: a generalization of a Theorem of Dobbs

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ABSTRACT. A commutative ring  $R$  is said to be a  $G$ -ring, in the sense of Adams, if  $T(R) = R[t^{-1}]$ , for some regular element  $t \in R$ , where  $T(R)$  denotes the total quotient ring of  $R$ . We first investigate the transfer of the  $G$ -property among pairs of rings sharing an ideal. Then, for  $A \subset B$  a couple of rings, we establish necessary and sufficient conditions for  $(A, B)$  to be a  $G$ -ring pair: that is, each intermediate ring  $A \subseteq R \subseteq B$  is a  $G$ -ring. In fact, we generalize a Theorem of D. Dobbs, characterizing  $G$ -domain pairs, to pairs of rings with zero divisors.

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## A covering condition for primary spectrum

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ABSTRACT. Let  $Prim(R)$  denote the set of primary ideals of a commutative ring  $R$ . The Zariski topology on  $Prim(R)$  is defined to be the topology whose closed sets are of the form  $V_{rad}(I)$ , denoting the set of primary ideals of  $R$  such that their radicals contain  $I$ . This topological space is called primary spectrum of  $R$ . In our study we examine the basis and some topological features of this space. In literature, commutative rings with the property that every ideal contained in the union of a family of prime ideals is contained in one of the primes of the family were examined. Analogous with this property, we also investigate under which condition a ring  $R$  satisfies the property that if  $X_r \subseteq \bigcup_{\alpha \in \Lambda} X_{s_\alpha}$  where  $r, s_\alpha \in R$  ( $\alpha \in \Lambda$ ) are nonzero elements of  $R$ , then  $X_r \subseteq X_{s_\alpha}$  for some  $\alpha \in \Lambda$ .

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## Commutator having idempotent values with automorphism in semiprime rings

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ABSTRACT. In the present paper it is shown that a semiprime ring  $R$  with characteristics different from 2 and 3 contains a nonzero central ideal of  $R$ , if  $R$  admits an automorphism  $\sigma$  such that  $[x^\sigma, y]^m = [x^\sigma, y]$  for all  $x, y \in R$ , where  $m > 1$  is a fixed positive integer. We also discuss the case when  $R$  is a prime and  $L$  is a noncentral Lie ideal of  $R$ . This result is in the spirit of the Herstein's theorem (commutator having idempotent values on rings).

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Mathematics Subject Classification (2010): 16N60, 16W20, 16R50.

Keywords: Prime and semiprime ring, automorphism, maximal right ring of quotient, generalized polynomial identity(GPI)

## On (Cofinitely) weak $\text{rad}-\oplus$ -supplemented modules

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ABSTRACT. This paper deals the property of weak  $\text{Rad}-\oplus$ -supplemented module. The class of weak  $\text{Rad}-\oplus$ -supplemented module lies between the class of  $\text{Rad}-\oplus$ -supplemented and  $\text{Rad}$ -supplemented modules. Moreover we study the properties of weak  $\text{Rad}-\oplus$ -supplemented and cofinitely weak  $\text{Rad}-\oplus$ -supplemented modules over some special kind of rings.

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Mathematics Subject Classification (2010): 16D10, 16D80.

Key words:  $\text{Rad}$ -supplemented; weak  $\text{Rad}-\oplus$ -supplemented module; Cofinitely weak  $\text{Rad}-\oplus$ -supplemented module.



## An ultrametric space of valuation domains of the field of rational functions

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ABSTRACT. Let  $V$  be a valuation domain of rank one and quotient field  $K$ . In this talk we study a class of valuation domains of the field of rational functions  $K(X)$  which lie over  $V$  and are indexed by the elements of  $\widehat{K}$ , the algebraic closure of the  $v$ -adic completion  $\widehat{K}$  of  $K$ . More precisely, let  $\widehat{V}$  be the integral closure of the completion  $\widehat{V}$  of  $V$  in  $\widehat{K}$ ; then, given  $\alpha \in \widehat{K}$ , the valuation domains we are interested in are of the form  $W_\alpha = \{\varphi \in K(X) \mid \varphi(\alpha) \in \widehat{V}\}$ . We give a necessary and sufficient condition for a valuation domain of  $K(X)$  to be of this form in the case when  $V$  is a discrete (DVR). Finally, we show that the space  $\{W_\alpha \mid \alpha \in \widehat{K}\}$  endowed with the Zariski topology is homeomorphic to the set of irreducible polynomials over  $\widehat{K}$  endowed with an ultrametric distance introduced by Krasner. Among other things, we will show how these valuation domains are important in the study of rings of integer-valued polynomials.

## Some conditions under which near-rings are rings

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ABSTRACT. In the present paper, we investigate the notion of generalized semiderivation satisfying certain algebraic identities in 3-prime near-ring  $N$  which forces  $N$  to be a commutative ring. Moreover, an example proving the necessity of the primeness of  $N$  is given.

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## Perinormal rings with zero-divisors

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ABSTRACT. In their recent papers *J. Algebra* 451 (2016) and arXiv:1511.06473v2, [math.AC], 29 Apr 2016, N. Epstein and J. Shapiro introduced and studied the perinormal domains: those domains  $A$  whose overrings satisfying going down over  $A$  are flat  $A$ -modules. We study the perinormal concept in the setup of rings with zero-divisors. We extend several results from perinormal domains case, for instance we prove that a Krull ring is perinormal. (Joint work with Tiberiu Dumitrescu.)

## **Identities with additive mappings in rings**

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**ABSTRACT.** Let  $R$  will be an associative ring,  $Z(R)$  the center of  $R$ ,  $Q$  its Martindale quotient ring and  $U$  its Utumi quotient ring. The center of  $U$ , denoted by  $C$ , is called the extended centroid of  $R$ . For  $x, y \in R$  and we set  $[x, y]_0 = x$ ,  $[x, y]_1 = xy - yx$  and inductively  $[x, y]_k = [[x, y]_{k-1}, y]$  for  $k > 1$ . The ring  $R$  is said to satisfy an Engel condition if there exists a positive integer  $k$  such that  $[x, y]_k = 0$  for all  $x, y \in R$ . Notice that an Engel condition is a polynomial  $[x, y]_k = \sum_{i=0}^k (-1)^i \binom{k}{i} y^i x y^{k-i}$  in non-commutative indeterminates  $x, y$ . Recall that a ring  $R$  is prime if  $xRy = \{0\}$  implies either  $x = 0$  or  $y = 0$ , and  $R$  is semiprime if  $xRx = \{0\}$  implies  $x = 0$ . An additive mapping  $d : R \rightarrow R$  is called a derivation if  $d(xy) = d(x)y + yd(x)$  holds for all  $x, y \in R$ . In particular  $d$  is an inner derivation induced by an element  $q \in R$ , if  $d(x) = [q, x]$  holds for all  $x \in R$ . By a generalized inner derivation on  $R$ , one usually means an additive mapping  $F : R \rightarrow R$  if  $F(x) = ax + xb$  for fixed  $a, b \in R$ . For a such a mapping  $F$ , it is easy to see that  $F(xy) = F(x)y + x[y, b] = F(x)y + xI_b(y)$ . This observation leads to the following definition : an additive mapping  $F : R \rightarrow R$  is called generalized derivation associated with a derivation  $d$  if  $F(xy) = F(x)y + xd(y)$  for all  $x, y \in R$ . In the present talk, we investigate the commutativity of  $R$  satisfying certain properties on some appropriate subset of  $R$ . We also examine the case where  $R$  is a semiprime ring.

## Sur la structure du groupe $\text{Gal}(\mathbb{k}_2^{(2)}/\mathbb{k})$ pour certains corps quadratiques réels $\mathbb{k}$

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RÉSUMÉ. Soient  $p_1 \equiv p_2 \equiv -q \equiv 1 \pmod{4}$  des nombres premiers. Posons  $\mathbb{k} = \mathbb{Q}(\sqrt{p_1 p_2 q})$ , et désignons par  $C_{\mathbb{k},2}$  son 2-groupe de classes et par  $\mathbb{k}_2^{(2)}$  son deuxième 2-corps de classes de Hilbert. Dans ce papier, on s'intéresse à déterminer le type de  $C_{\mathbb{k},2}$  et la structure du groupe  $G = \text{Gal}(\mathbb{k}_2^{(2)}/\mathbb{k})$  en utilisant la capitulation des 2-classes de  $\mathbb{k}$  dans ses extensions quadratiques non-ramifiées.

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## Biordered sets from involution rings and projection lattice

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ABSTRACT. In [5] K.S.S. Nambooripad introduced the concept of a biordered set as a partial algebra  $(E, \omega^r, \omega^l)$  where  $\omega^r$  and  $\omega^l$  are two quasiorders on  $E$  satisfying certain axioms to study the structure of a regular semigroup. Later on it is also shown that every biordered set arises as the set idempotents of some semigroup thus biordered set turns out to be a major tool in the study of semigroups. It is also well known that when commutes the orders  $\omega^r$  and  $\omega^l$  coincides and the biordered set reduces to a partially ordered set. In this paper we extend the biordered set approach to the study of the structure of a ring, by describing the biordered sets (both additive and multiplicative) arising from the ring. Further it is shown that in certain rings with involution, when idempotents commutes the projections form a complemented distributive lattice.

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## Maximal codes

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ABSTRACT. The theory of coding is the study of methods allowing the transfer Information effectively. But for the retrieval of information always, one seeks to minimize time by the use of algorithms whose goal is to reduce the exhaustive search.

In this work we try to introduce the concept of Maximal codes that are built over rings, more precisely we will give Maximal codes for special rings, Namely that the notion of maximal codes has been used by Christophe Chapote, these maximal codes are constructed over finite fields, and these codes are used for coding and decoding by minimizing the time.

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## On monogeneity of cubic cyclic extension

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ABSTRACT. In this paper, with a given integral basis of  $O_K$ : the integral closure of unramified cyclic cubic number field  $K = \mathbb{Q}(\beta)$ , we explicit When  $O_K$  is monogenic. As a consequence of this theoretical result the index  $I_{O_K}(\beta)$  of  $K$  has straightforwardly computed. Furthermore we test if  $\beta$  is a generator of normal integral basis of  $K$ .

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## Generalized $(\in, \in \vee q_k)$ -Fuzzy subsemigroups and ideals in semigroups

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ABSTRACT. The main motivation of this article is to generalize the concept of fuzzy ideals,  $(\alpha, \beta)$ -fuzzy ideals,  $(\in, \in \vee q_k)$ -fuzzy ideals of semigroups. By using the concept of  $q_k^\delta$ -quasi-coincident of a fuzzy point with a fuzzy set, we introduce the notions of  $(\in, \in \vee q_k^\delta)$ -fuzzy left ideal,  $(\in, \in \vee q_k^\delta)$ -fuzzy right ideal of a semigroup. Special sets, so called  $Q_k^\delta$ -set and  $[\lambda_k^\delta]_t$ -set, condition for the  $Q_k^\delta$ -set and  $[\lambda_k^\delta]_t$ -set to be left (resp. right) ideals are considered. We finally characterize different classes of semigroups (regular, left weakly regular, right weakly regular) in term of  $(\in, \in \vee q_k^\delta)$ -fuzzy left ideal,  $(\in, \in \vee q_k^\delta)$ -fuzzy right ideal and  $(\in, \in \vee q_k^\delta)$ -fuzzy ideal of semigroup  $S$ .

## Some properties of $\star$ -prime rings

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**ABSTRACT.** The main purpose of this work is to provide some properties of  $\star$ -prime rings with involution  $\star$ . First, we study the relationship between the primeness and the  $\star$ -primeness of rings. Afterwards, we investigate the semiprimeness of a  $\star$ -prime rings. At the end, we characterize the left centralizers in  $\star$ -prime rings.

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Mathematics Subject Classification (2010): .

Key words: Prime rings, semiprime ring, involution,  $\star$ -prime ring,  $\star$ -ideal, left centralizer.

## Construction of a strongly co-hopfian abelian which the torsion part isn't strongly co-hopfian

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ABSTRACT. An abelian group  $A$  is called strongly co-hopfian if for every endomorphism  $\alpha$  of  $A$  the chain  $Im(\alpha) \supseteq Im(\alpha^2) \supseteq Im(\alpha^3) \supseteq Im(\alpha^4) \supseteq \dots$  is stationary. In this work we characterize some properties of the strongly co-hopfian abelian group. Then we show that the  $p$ -component of strongly co-hopfian abelian group is also strongly co-hopfian but for the torsion part we construct strongly co-hopfian abelian group whose the torsion part is not strongly co-hopfian.

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Key words and phrases: strongly co-hopfian, Abelian groups,  $p$ -group, order, direct sums of cyclic groups, basic subgroups, monomorphism group, automorphism group.

## Anneaux pour lesquels la réciproque du Lemme de Schur est vérifiée

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RÉSUMÉ. Si  $M$  est un module simple d'un anneau  $R$  alors  $End_R(M)$  est un anneau à division (Lemme de Schur). La réciproque de ce résultat que nous appelons la propriété CSL n'est pas toujours vraie. Notre objet est de communiquer cette réciproque pour quelques classes d'anneaux : anneaux noetheriens à gauche, anneaux réguliers au sens de von Neumann (VNR) et anneaux parfaits. D'abord, nous établissons qu'un anneau noetherien à gauche  $R$  est un CSL-anneau si et seulement si  $R$  est un anneau artinian à gauche et primairement décomposable. Ensuite, nous montrons qu'un anneau (VNR) dont tous les quotients primitifs sont artiniens est un CSL-anneau. En particulier, tout anneau régulier à identité polynômiale vérifie la propriété CSL. Enfin, la propriété CSL pour un anneau parfait, nous montrons que  $R$  est un CSL-anneau si et seulement si  $R$  est un produit d'anneaux primaires.

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## Generating elliptic curves for cryptography

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ABSTRACT. Elliptic curve cryptography (ECC) [1, 2] is a very efficient technology for implementing public key cryptosystems and public key infrastructures (PKI), it offers encryption/decryption, digital signature, and key exchange solutions. During the last decade, many elliptic curve standards were proposed [3, 4]. However, these curves do not satisfy all the security and the efficiency requirements. In fact, securing cryptographic protocols while preserving efficiency is a big challenge for cryptographers in practice. In this paper, we discuss the compatibility of cryptographic requirements and how can we generate new families of elliptic curves over finite fields for cryptographic applications using SAGE [5] or PARI/GP [6] systems.

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## Epimorphisms and dominions

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ABSTRACT. Let  $\mathcal{C}$  be a category. A morphism  $\alpha$  of  $\mathcal{C}$  is an epimorphism if whenever  $\beta$  and  $\gamma$  are morphisms of  $\mathcal{C}$  such that  $\alpha\beta = \alpha\gamma$ , then  $\beta = \gamma$ . In a concrete category  $\mathcal{C}$ . It is obvious that any surjective morphism is epimorphism but the converse need not be true in general. In the category of semigroups epimorphisms are not necessarily onto, for example the inclusion  $i : (0, 1) \rightarrow (0, \infty)$  regarding both the intervals as multiplicative semigroups is an epimorphism.

Let  $U$  be a subsemigroup of a semigroup  $S$ . we say that  $U$  dominates  $d$  of  $S$  if whenever  $\alpha, \beta : S \rightarrow T$  are morphisms such that  $u\alpha = u\beta$  for all  $u \in U$ , then  $d\alpha = d\beta$ . The set of all elements of  $S$  dominated by  $U$  is called the Dominion of  $U$  in  $S$  and is denoted by  $\text{Dom}(U, S)$  which is a subsemigroup of  $S$  containing  $U$ . The concepts of epimorphisms and dominions are closely related as  $\alpha : S \rightarrow T$  is epimorphism if and only if the inclusion  $i : s^\alpha \rightarrow T$  is epi and the inclusion  $i : s^\alpha \rightarrow T$  is epi iff and only if  $\text{Dom}(S\alpha, T) = T$ . Suppose a semigroup  $S$  satisfies an identity  $I$ , it is natural too also does  $I$  be satisfied by the epimorphic image of  $S$ . In the present talk we shall be satisfied by the epimorphic image of  $S$ . In the present talk we shall try to answer this question.

## Left generalized multiplicative derivations and commutativity of 3-prime near-rings

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ABSTRACT. Let  $\mathcal{N}$  be a left near-ring. A map  $d : \mathcal{N} \rightarrow \mathcal{N}$  is called a multiplicative derivation of  $\mathcal{N}$  if  $d(xy) = xd(y) + d(x)y$  holds for all  $x, y \in \mathcal{N}$ . A map  $f : \mathcal{N} \rightarrow \mathcal{N}$  is called a right generalized multiplicative derivation of  $\mathcal{N}$  if there exists a multiplicative derivation  $d$  of  $\mathcal{N}$  such that  $f(xy) = xd(y) + f(x)y$  for all  $x, y \in \mathcal{N}$ . Here we say that  $f$  is a right generalized multiplicative derivation of  $\mathcal{N}$  with associated multiplicative derivation  $d$  of  $\mathcal{N}$ . Similarly a map  $f : \mathcal{N} \rightarrow \mathcal{N}$  is called a left generalized multiplicative derivation of  $\mathcal{N}$  if there exists a multiplicative derivation  $d$  of  $\mathcal{N}$  such that  $f(xy) = xf(y) + d(x)y$  for all  $x, y \in \mathcal{N}$ . The map  $f$  will be called a left generalized multiplicative derivation of  $\mathcal{N}$  with associated multiplicative derivation  $d$  of  $\mathcal{N}$ . Finally, a map  $f : \mathcal{N} \rightarrow \mathcal{N}$  will be called a generalized multiplicative derivation of  $\mathcal{N}$  if it is both a right as well as a left generalized multiplicative derivation of  $\mathcal{N}$  with associated multiplicative derivation  $d$  of  $\mathcal{N}$ . Note that if in the above definition both  $d$  and  $f$  are assumed to be additive mappings, then  $f$  is said to be a generalized derivation with associated derivation  $d$  of  $\mathcal{N}$ . In the present paper, we investigate the commutativity of 3-prime near-ring  $\mathcal{N}$  satisfying certain conditions and identities involving left generalized multiplicative derivations on semigroup ideals. Moreover, examples justifying the necessity of 3-primeness condition in all the results are provided. We have also constructed examples to justify the fact that the results proved here are not true for right generalized multiplicative derivations.

## On $n$ -absorbing ideals of power series rings

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ABSTRACT. Let  $R$  be a commutative ring with  $1 \neq 0$  and  $n$  a positive integer. The concept of 2-absorbing ideals was introduced by A. Badawi, in [2], as a generalization of prime ideals, and some properties of 2-absorbing ideals were studied. Precisely, a proper ideal  $I$  of  $R$  is said to be 2-absorbing if  $abc \in I$  for  $a, b, c \in R$  implies that  $ab \in I$  or  $ac \in I$  or  $bc \in I$ .

Later, in 2011, D. D Anderson and A. Badawi generalized the concept of 2-absorbing ideals to  $n$ -absorbing ideals [1]. According to their definition, a proper ideal  $I$  of  $R$  is called an  $n$ -absorbing ideal if whenever  $a_1 \dots a_{n+1} \in I$  for  $a_1, \dots, a_{n+1} \in R$ , then there are  $n$  of the  $a_i$ 's whose product is in  $I$ .

In this talk, we study Anderson-Badawi conjectures, the stability of 2-absorbing ideals in the power series rings and we give some basic properties of  $n$ -absorbing ideals in  $U$ -rings.

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## Tower formula of discriminant

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ABSTRACT. Let  $R$  be a commutative ring. Let  $A$  be a free  $R$ -algebra of finite rank and  $M$  a free  $A$ -module of finite rank. In this work we establish an interesting tower formula of discriminant of  $M$ . More precisely we prove that the discriminant of the bilinear module  $M$  over  $R$  is the product of the norm of discriminant of bilinear module  $M$  over  $A$  and some power of the classical discriminant of the  $R$ -algebra  $A$ . As an application we compute the discriminant of the algebra associated to a B-J polynomial, and some graded fields.

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## Monotonicity of finite Dirichlet's L function

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ABSTRACT. Dirichlet's L Function with nontrivial primitive character have region non zero. Finite Dirichlet's L function continuous and differentiable. This paper aim to show monotonicity Dirichlet's L Function by Logarithmically complete monotonicity.

## Diophantine equations associated Fibonacci numbers

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ABSTRACT. Let  $\{F_n\}$  denote the sequence of Fibonacci numbers defined by  $F_0 = 0$ ,  $F_1 = 1$ , and  $F_n = F_{n-1} + F_{n-2}$  for  $n \geq 2$ .

There exist no end of results linked different relations among Fibonacci numbers. Now assume that  $p$  and  $q$  are positive integers. The present talk studies the equations

$$F_1^p + 2F_2^p + \dots + kF_k^p = F_n^q$$

in the positive integer unknowns  $k$  and  $n$ .

The conjecture is that the nontrivial solutions are only

$$F_8 = 21 = F_1 + 2F_2 + 3F_3 + 4F_4,$$

$$F_4^2 = 9 = F_1 + 2F_2 + 3F_3,$$

$$F_4^3 = 27 = F_1^3 + 2F_2^3 + 3F_3^3.$$

We solve completely the cases  $p = q \in \{1, 2\}$ .

## Le Théorème de Batman sur la fonction PHI d'Euler

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RÉSUMÉ. Soit  $\varphi$  la fonction arithmétique d'Euler définie par  $\varphi(n) = \sum_{1 \leq m \leq n, (m,n)=1} 1$ , avec  $m$  et  $n$  des entiers naturels, on désigne par  $\phi(x)$  le nombre des entiers naturels  $n$  tel que  $\phi(n) \leq x$ , c'est-à-dire

$$\phi(x) = \sum_{\varphi(n) \leq x} 1$$

Il est connu [Erdős] que  $\phi(x) \simeq Ax$ , ( $x \rightarrow +\infty$ ), où  $A$  une constante effective tel que

$$A = \frac{\zeta(2)\zeta(3)}{\zeta(6)} \sim 1,9435964\dots$$

Le premier qui a étudié le reste  $R(x) = \phi(x) - Ax$  c'est P. Bateman en 1972, le résultat obtenu est le suivant : Pour  $x > \exp(10^4)$  et  $0 < \theta < 1$ , on a

$$\phi(x) = Ax + O\left(x \exp\left(\frac{-\theta}{\sqrt{2}} \sqrt{(\ln x)(\ln \ln x)}\right)\right).$$

La démonstration de ce résultat est basée sur l'intégration sur un contour du plan complexe à l'aide du théorème des résidus, en utilisant la formule de Perron .

En 1992, Smati a exploré les possibilités offertes par une méthode élémentaire et obtenait la première estimation effective de  $R(x)$ , pour  $x \geq 3$

$$|R(x)| \leq 1,4x(\ln x)(\ln \ln x) \times$$

$$\exp\left\{-\left(1 - \frac{\ln(\ln(\ln x)) + 4 - \ln 2}{\ln \ln x}\right) \sqrt{(1/2)(\ln x)(\ln \ln x)}\right\}$$

La dernière estimation effective du reste  $R(x)$  est obtenue en 2009 par A.Derbal, pour  $x \geq 240$

$$|R(x)| \leq 58,61x \exp\left(-(\sqrt{2}/8)\sqrt{(\ln x)(\ln \ln x)}\right)$$

Dans un travail future, on compte d'améliorer la constante de cette dernière majoration.

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## Sur le deuxième $l$ -groupe de classes de certains corps de nombres de type $(l, l)$ et applications

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RÉSUMÉ. Soient  $l$  un nombre premier,  $F$  un corps de nombres et  $k$  une extension de  $F$  de degré  $l$ . On suppose que le  $l$ -groupe de classes de  $F$ ,  $\text{Cl}_l(F)$ , est trivial et que le conducteur de  $k|F$  est divisible exactement par deux premiers  $\Pi_1$  et  $\Pi_2$ . Alors la structure du groupe  $\text{Gal}\left((k^*)^{(l)}|F\right)$ , où  $k^* = (k|F)^*$  est le corps de genre relatif de l'extension  $k|F$ , est complètement déterminé, une telle structure est liée à la principalisation des premiers  $\Pi_1$  et  $\Pi_2$ . Si de plus le  $l$ -rang du groupe de classes de  $k^*$  est inférieur à 2, alors le deuxième  $l$ -groupe de classes est déterminé.

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## Second 3-class groups of parametrized real quadratic fields

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ABSTRACT. For certain real quadratic fields  $K = \mathbb{Q}(\sqrt{d})$  of 3-class rank  $\varrho_3(K) = 2$  and discriminant  $0 < d < 10^9$ , given in terms of parameters  $d = 4uw^3 - 27u^2$  by Kishi and Miyake, the Galois group  $G = \text{Gal}(\mathbb{F}_3^2(K)|K)$  of the second Hilbert 3-class field  $\mathbb{F}_3^2(K)$  of  $K$  and the 3-principalization type  $\varkappa(K)$  of  $K$  are determined.

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## Note on the weak global dimension of coherent bi-amalgamations

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ABSTRACT. Let  $f : A \rightarrow B$  and  $g : A \rightarrow C$  be two ring homomorphisms and let  $J$  and  $J'$  be two ideals of  $B$  and  $C$ , respectively, such that  $f^{-1}(J) = g^{-1}(J')$ . The bi-amalgamation of  $A$  with  $(B, C)$  along  $(J, J')$  with respect to  $(f, g)$  is the subring of  $B \times C$  given by

$$A \bowtie^{f,g} (J, J') = \{(f(a) + j, g(a) + j') / a \in A, (j, j') \in J \times J'\}.$$

In this talk, we discuss the weak global dimension of coherent bi-amalgamations.

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## On the flatness of $\text{Int}(E, D)$ as a $D$ -module

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ABSTRACT. Let  $D$  be an integral domain with quotient field  $K$ ,  $E$  a subset of  $K$  and  $X$  an indeterminate over  $K$ . The set of *integer-valued polynomials on  $E$*  is defined by  $\text{Int}(E, D) = \{f \in K[X] \mid f(E) \subseteq D\}$ . Clearly,  $\text{Int}(E, D)$  is a subring of  $K[X]$  and if  $E = D$ , then  $\text{Int}(E, D) = \text{Int}(D)$ , the ring of integer-valued polynomials on  $D$ . These two rings,  $\text{Int}(D)$  and  $\text{Int}(E, D)$ , were studied extensively for a long time and much is known about them. [2] is a good reference on the algebraic properties of the rings of integer-valued polynomials. In this paper, we present some progress in the study of when  $\text{Int}(E, D)$  is locally free, or flat, as a  $D$ -module (cf. [3, Problem 19]).

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## Some sufficient conditions for $M$ -hypercyclicity of $C_0$ -semigroup

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ABSTRACT. The goal in this paper is to give a sufficient conditions for a  $C_0$ -semigroup acting on complex separable infinite dimensional Banach space  $X$  to be  $M$ -hypercyclic, this is the  $M$ -hypercyclicity criterion. Moreover we characterize the  $C_0$ -semigroup satisfying this criterion.

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Key words:  $C_0$ -semigroups, Hypercyclicity, Topologically transitive,  $M$ -hypercyclicity,  $M$ -transitive.

## On dual Baer modules and a generalization of dual Rickart modules

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ABSTRACT. We study the notion of *wd-Rickart* (or *weak dual Rickart*) modules (i.e. modules  $M$  such that for every nonzero endomorphism  $\varphi$  of  $M$ , the image of  $\varphi$  contains a nonzero direct summand of  $M$ ). We obtain two new characterizations of dual Baer modules. We also characterize the class of rings  $R$  for which every right  $R$ -module is wd-Rickart. A number of examples which delineate the concepts and results are included.

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## Stability of Gorenstein $gr$ -flat modules

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ABSTRACT. In this paper, first we introduce second degree Gorenstein  $gr$ -flat modules. Secondly, we introduce  $GF$ - $gr$ -closed rings and gives a characterization of this ring. Finally, we show that the two-degree Gorenstein  $gr$ -flat modules are nothing more than that the Gorenstein  $gr$ -flat modules over a  $GF$ - $gr$ -closed ring.

## Semicommutativity of the rings relative to prime radical

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**ABSTRACT.** We introduce a class of rings which behave like semicommutative rings by employing the prime radical of a ring. This kind of rings is called  $P$ -semicommutative. We investigate general properties of  $P$ -semicommutative rings and some interrelations among  $P$ -semicommutative rings and the other versions of semicommutativity, such as weakly semicommutative rings, nil-semicommutative rings and central semicommutative rings. It is proved that the concepts of clean rings and exchange rings coincide for  $P$ -semicommutative rings. A relation between maximal right ideals and idempotents of a  $P$ -semicommutative ring is obtained. Characterizations of  $P$ -semicommutative rings with their extensions are also given.

## On generalization of Schur's Lemma for group representation on module over PIDs

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**ABSTRACT.** Group representations allow one to study an abstract group in terms of linear transformations on vector spaces. The main point of studying group representations is to reduce group theoretic problems to those of linear algebra which are well understood. The inverse direction can also be fruitful; that is, sometimes one can understand certain problems of linear algebra better by using ideas from group representation theory. One of the tool in studying the group representation is the Schur's Lemma which state that if  $V$  and  $W$  are irreducible representations of group  $G$  over field  $k$ , then (1) If  $\phi : U \rightarrow V$  is a  $G$ -module homomorphism, then either  $Im(\phi) = \{0\}$  or  $\phi$  is an isomorphism. (2) If  $\phi : U \rightarrow V$  is a  $G$ -module isomorphism, then there exists  $\lambda \in \mathbb{C}$  such that  $\phi(v) = \lambda v$  for all  $v \in V$ .

In this paper we will present the generalization of the Schur's Lemma for group representation on module over PIDs. The result of this study will be used to investigate the complete reducible properties of group representation on module over PIDs as generalization of group representation on vector spaces over field.

## On commutativity of rings and Banach algebras with generalized derivations

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ABSTRACT. The objective of this paper is to discuss the commutativity of a prime ring  $R$  with centre  $Z(R)$ , which admits a generalized derivation  $f$  associated with a non zero derivation  $d$  such that  $f([x^m, y^n]) \pm [x^m, y^n] \in Z(R)$  for all  $x, y \in R$ . Finally, we apply these purely ring theoretic results to obtain commutativity of Banach algebra. In particular, we prove that if  $A$  is a prime Banach algebra which admits a continuous linear generalized derivation  $f$  associated with a nonzero continuous linear derivation  $d$  such that either  $f([x^m, y^n]) - [x^m, y^n] \in Z(A)$  or  $f([x^m, y^n]) + [x^m, y^n] \in Z(A)$ , for an integer  $m = m(x, y) > 1$  and sufficiently many  $x, y$  in  $A$ , then  $A$  is commutative.

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Key words: Prime ring; Banach algebra; Derivations; generalized derivation.

## Functionals on $\mathbb{R}$ -vector spaces

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ABSTRACT. In this study we introduce the notion of functionals on  $\mathbb{R}$ -vector spaces and obtain various properties. We also introduce the concept of dual spaces and Inner product in  $\mathbb{R}$ -vector spaces and study their properties.

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Mathematics Subject Classification (2010): .

Key words:  $\mathbb{R}$ -vector space,Special homomorphism,Functional,Dual Space,Inner Product.

## On commutativity of rings and Banach algebras with generalized derivations

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ABSTRACT. Throughout this paper  $R$  is an associative ring with identity, and  $R\text{-Mod}$  denotes the category of all the unitary left  $R$ -modules. The category of  $M$ -subgenerated modules (the Wisbauer category) is denoted by  $\sigma[M]$  (see [2]). In [1], Raggi et al. defined the notion of prime submodules. Let  $M \in R\text{-Mod}$ . A submodule  $N$  of  $M$  is called *fully invariant* if  $f(N) \leq N$  for each  $R$ -homomorphism  $f : M \rightarrow M$ . Let  $M \in R\text{-Mod}$  and let  $N \neq M$  be a fully invariant submodule of  $M$ . The submodule  $N$  is said to be *prime in  $M$*  if whenever  $K, L$  are fully invariant submodules of  $M$  with  $K \cdot L \leq N$ , then  $K \leq N$  or  $L \leq N$ .

A *preradical* over the ring  $R$  is a subfunctor of the identity functor on  $R\text{-Mod}$ . Denote by  $R\text{-pr}$  the class of all preradicals over  $R$ . For  $\sigma, \tau \in R\text{-pr}$  and  $M \in R\text{-Mod}$ , we define  $(\sigma\tau)(M) = \sigma(\tau M)$ . The notion of prime preradicals is defined in In [1]. Let  $\sigma \in R\text{-pr}$ .  $\sigma$  is called *prime in  $R\text{-pr}$*  if  $\sigma \neq 1$  and for any  $\tau, \eta \in R\text{-pr}$ ,  $\tau \eta \preceq \sigma$  implies that  $\tau \preceq \sigma$  or  $\eta \preceq \sigma$ .

In this talk we discuss on generalizations of the notions of prime preradicals and prime submodules. The preradical  $\sigma \in R\text{-pr}$  is called *2-absorbing* if  $\sigma \neq 1$  and, for each  $\eta, \mu, \nu \in R\text{-pr}$ ,  $\eta\mu\nu \preceq \sigma$  implies that  $\eta\mu \preceq \sigma$  or  $\eta\nu \preceq \sigma$  or  $\mu\nu \preceq \sigma$ . We will denote by  $R\text{-Ass}$  the class of all  $R$ -modules  $M$  that the operation  $\alpha$ -product is associative over fully invariant submodules of  $M$ , i.e., for any fully invariant submodules  $K, N, L$  of  $M$ ,  $(K \cdot N) \cdot L = K \cdot (N \cdot L)$ . Let  $M \in R\text{-Ass}$  and let  $N \neq M$  be a fully invariant submodule of  $M$ . The submodule  $N$  is said to be *2-absorbing in  $M$*  if whenever  $J, K, L$  are fully invariant submodules of  $M$  with  $J \cdot K \cdot L \leq N$ , then  $J \cdot K \leq N$  or  $J \cdot L \leq N$  or  $L \cdot K \leq N$ .

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## On weakly prime and weakly semiprime ideals of commutative rings

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Dedicated to Our Professor El Amin KAIDI

ABSTRACT. Let  $R$  be a commutative ring with identity and let  $P$  be a proper ideal of  $R$ . The notion of weakly prime (resp., weakly semiprime) ideals is introduced by Anderson-Smith (resp., by Badawi), and considered as a generalization of prime (resp., semiprime) ideals. Recall that an ideal  $P$  is called weakly prime (resp., weakly semi-prime) if  $0 \neq ab \in P$  implies  $a \in P$  or  $b \in P$  (resp.,  $0 \neq a^2 \in P$  implies  $a \in P$ ).

In this paper, we investigate the stability of the weakly prime and weakly semiprime ideals under the amalgamated duplication.

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## The uniqueness of complete norm topology in Banach-Jordan pairs

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ABSTRACT. The aim of this paper consists in proving the automatic continuity of a homomorphism  $\varphi = (\varphi_+, \varphi_-)$  defined from a Banach-Jordan pair  $V = (V^+, V^-)$  [4] onto a semisimple Banach-Jordan pair  $W = (W^+, W^-)$ . As a direct consequence, we show that every semisimple Banach-Jordan pair  $V = (V^+, V^-)$  over  $\mathbb{R}$  or  $\mathbb{C}$  has the uniqueness of norm property, that is we show that if  $V = (V^+, V^-)$  is a semisimple Banach-Jordan pair with each of the norms  $\|\cdot\|_+$ ,  $\|\cdot\|'_+$ ,  $\|\cdot\|_-$ ,  $\|\cdot\|'_-$  then these norms define the same topologies respectively in  $V^+$  and  $V^-$ . The analogous results earlier obtained by B. E. Johnson with respect to Banach algebras [3] and by B. Aupetit with respect to Banach-Jordan algebras [1] become direct consequences of the main result in this paper.

Unlike Johnson's and Aupetit's procedures based respectively on Representation Theory and the subharmonicity of spectral radius, our approach consists in using Spectral Theory in Banach-Jordan pairs to prove that the separating subspace  $S(\varphi) = (S(\varphi_+), S(\varphi_-))$  of the homomorphism  $\varphi = (\varphi_+, \varphi_-)$  is contained in the Jacobson radical  $Rad(W) = (Rad(W^+), Rad(W^-))$  of  $W = (W^+, W^-)$  [4]. It follows, by means of the Graph Theorem [2], that  $\varphi_\sigma$  ( $\sigma = \pm$ ) is continuous. The uniqueness of norm topology is settled by considering the identities  $Id_{V^\sigma} : (V^\sigma, \|\cdot\|_\sigma) \rightarrow (V^\sigma, \|\cdot\|'_\sigma)$  which are actually continuous and arguing with  $\|\cdot\|_\sigma$  and  $\|\cdot\|'_\sigma$  interchanged.

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## A special chain theorem in the set of intermediate rings

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ABSTRACT. Let  $R \subset S$  be an extension of integral domains, and let  $R^*$  be the integral closure of  $R$  in  $S$ . Our main goal is to study  $[R, S]$ , the set of intermediate rings between  $R$  and  $S$ . As a main tool, we establish an explicit description of any intermediate ring in terms of localizations of  $R$  (or  $R^*$ ). This study effectively enables us to characterize the minimal extensions in  $[R, S]$  and we prove a special chain theorem concerning the length of an arbitrary maximal chain in  $[R, S]$ . Also we establish several necessary and sufficient conditions for which every ring contained between  $R$  and  $S$  compares with  $R^*$  under inclusion, this answers a key question that figured in the work of Gilmer and Heinzer [*Intersections of quotient rings of an integral domain*], J.Math. Kyoto Univ. 7 (1967), 133-150].

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Key words: Intermediate ring; Minimal ring extension; Finite chain condition; Maximal chain; Normal pair; Support; Conductor.

Capitulation of the 2-ideal classes of the field  $\mathbb{Q}(\sqrt{2q_1q_2}, i)$  of type  
(2, 2)

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ABSTRACT. Let  $q_1 \equiv q_2 \equiv -1 \pmod{4}$  be two different prime integers and  $i = \sqrt{-1}$ . Put  $\mathbb{k} = \mathbb{Q}(\sqrt{2q_1q_2}, i)$ , and denote by  $\mathbf{Cl}_2(\mathbb{k})$  the 2-part of its class group  $\mathbf{Cl}(\mathbb{k})$ . Let  $\mathbb{k}_1^{(2)}$  be the Hilbert 2-class field of  $\mathbb{k}$ ,  $\mathbb{k}_2^{(2)}$  the Hilbert 2-class field of  $\mathbb{k}_1^{(2)}$  and  $G = \text{Gal}(\mathbb{k}_2^{(2)}/\mathbb{k})$  be the Galois group of  $\mathbb{k}_2^{(2)}/\mathbb{k}$ . Assume  $\mathbf{Cl}_2(\mathbb{k}) \simeq (2, 2)$ ; the aim of this note is to study the capitulation of the 2-ideal classes of  $\mathbb{k}$  in the three unramified extensions of  $\mathbb{k}$  within  $\mathbb{k}_1^{(2)}$ , and to determine the structure of  $G$ .

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Key words: fundamental systems of units, 2-class group, capitulation, quadratic fields, biquadratic fields.

## Three pearls of Bernoulli numbers

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ABSTRACT. The Bernoulli numbers are fascinating and ubiquitous numbers; they occur in several domains of Mathematics like Number theory (FLT), Group theory, Calculus and even in Physics. Since Bernoulli's work, they are yet studied to find their secret [?], particularly to find relationships between them. In this talk, we give, firstly, a short response [?] to a problem asked, in 1971, by Carlitz [3] and studied by many authors like Prodinger [7], the second pearl is an answer to a question asked, in 2008, by Tom Apostol [1]. The third pearl is a new proof of a relationship already given in 2011 [11].

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Key words: Bernoulli numbers, Bernoulli polynomials.

## Modules over a new Ring of ponderation functions

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ABSTRACT. In this paper we study a class of modules over a new ring of ponderation functions recently introduced in [1], so we prove that Laplace transform and Fourier transform generate some free modules over the ring of ponderation functions. Moreover we characterize the projective modules and simple modules and we prove that the socle of this ring is not an injective module.

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## Caractérisation des signaux fECG par le microscope mathématique : Transformée en ondelettes

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**RÉSUMÉ.** Les maladies et malformations cardiaques sont les principales causes de décès à la naissance. Chaque année environ un bébé sur 125 présente une forme de malformations cardiaques congénitales qui apparaissent dans les premières semaines de grossesse, le suivi régulier de la fréquence cardiaque fœtale et la détection précoce des anomalies aide le cardio-pédiatre à prescrire les médicaments appropriés même pendant la grossesse et à prendre les précautions adaptées. L'ECG du fœtus est la représentation temporelle de l'évolution du champ électrique dans le muscle cardiaque permettant de détecter le risque. Malheureusement le signal fECG de faible énergie se trouve noyé dans celui de la mère de puissance beaucoup plus forte. Notre objectif est de caractériser l'ECG du fœtus à partir du signal global en utilisant une approche avancée se basant sur un microscope mathématique appelée Transformée en ondelettes.

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